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# A MATHEMATICAL STUDY OF COVID-19 EPIDEMICS: THE ROLE OF FEAR OF INFECTION

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**Abstract:** The fear associated with an infectious disease plays a major role in the management of the infectious disease. Thus, this study tried to assess the impact of fear of infection on the spread of COVID-19 using a mathematical modelling approach. The model considered was shown to have two equilibria points, namely, the COVID-19 free equilibrium (CFE) and the COVID-19 persistent equilibrium (CPE). The computed reproduction number ( $R_{C_1}$) was used to validate the local stability of the CPE whenever $R_{C_1}$ is above one while the CFE is globally asymptotically stable $R_{C_1}$ □1. Next, we show that the condition $R_{C_1}$ □1 is sufficient to halt the spread of COVID-19 by showing that the model possesses forward bifurcation. The sensitivity analysis suggests that the fear of infection does not influence the $R_{C_1}$ while numerical simulation indicates that the population of all infected humans, COVID-19 deceased individuals and the concentration of the COVID-19 viruses in the environment increases as the fear of infection (□$_1$) reduces.

**Keywords:** COVID-19; Mathematical Model, Fear of Infection; Lyapunov Function; Bifurcation Analysis

## 1.0 Introduction

Recently, several studies were conducted by researchers to investigate the influence of fear of the dynamics of some real-life phenomena. These areas include but are not limited to fear of crime (Benavente et al., 2023; Bove et al., 2023), fear of terrorism (Altier et al., 2023; Kaskeleviciute et al., 2023; Falcoˆ-Gimeno et al., 2023), fear of infection (Tang et al., 2023; Colonnello et al., 2023; Kroesen et al., 2023), fear of vaccination (Amanna & Slifka, 2005; Sato & Fintan, 2020; Malas et a., 2021; Bendau et al., 2021). To gain better insight into how fear impacts the dynamics of these real-life problems, mathematical modelling is used. Terrorism model (Okoye et al., 2023), ecological models (Mondal et al., 2023; Pratama et al., 2023; Zhao & Shao, 2023), and epidemic models ( Juga et al., 2021; Yousef et al., 2023; Rashid & Jarad, 2023) are some of the mathematical models that captures the impact of fear. Murad et al. (2022), pointed out that one of the factors that affects the management of COVID-19 is the fear of being infected (Murad et al., 2022). Although the fear of the pandemic has been established by researchers to trigger various mental health issues someone who is scared of being infected may not be in a stable frame of mind (Murad et al., 2022). Apart from psychological damage caused by the pandemic, the living conditions of individuals in the populace are affected (RodrAguez-

Hidalgo et al., 2020), and fear of the pandemic may weaken the public health delivery system (Shanafelt, et al., 2020). Nevertheless, there is a positive correlation between the fear of COVID-19 infection and strict compliance with preventive measures and regulations which could be beneficial to public health services (Ilesanmi & Afolabi, 2020; Murad et al., 2022; Karlsson et al., 2021; Recio-Vivas et al., 2020). Thus, the need to measure correctly the impact of fear of COVID-19 infection is of great importance to disease management. Several mathematical models describe the spread of COVID-19 infection (Elmojtaba et al., 2023; Odetunde et al., 2022; Johnson and Pell, 2022; Thirthar et al., 2023; Usman et al., 2023). In particular, Usman et al. (2023), proposed a mathematical model that governs the transmission dynamics of COVID-19 infection coupled with the fear of infection. The proposed model was solved using the non-standard finite difference method but failed to assess the impact of fear of infection qualitatively. Thus, the goal of this paper is to study the impact of fear of infection on COVID-19 by analyzing the model presented by Usman et al. (2023) as follows:

dS = −P □S − K S V □□ dt $_1$ +□$_2$

□ dV = □ dt □$_1$S −e V□ −K V$_2$ □ dE =□(S +eV )− K E$^3$ □□ dt □ dQ □

dt =□$_1$E − K Q$_4$ □□ dA □ dt =□$_1$E − K A$_5$ □□

dI =□ A−K I$_6$ □□ dt $_2$ □ dH □ dt =□ □$_3$Q+ $_2$I −K H$_7$ □□ dR □

dt =□ □ □ □$_1$A+ $_2$I + $_3$H − R□□ dD □

dt = l A l I$^1$ + +$_2$ l H$_3$ −□D □□ (1.1) dW

I □ □H − W□

=□ +□

dt □□

Where S t( ) is the unvaccinated compartment, V t( ) is the vaccinated susceptible compartment, E t( ) is the exposed compartment, Q t( ) is the quarantined compartment, A t( ) is the asymptomatic compartment, I t( ) is the symptomatic compartment, H t( ) is the hospitalized compartment, R t( ) is the recovered compartment, D t( ) is the dead compartment, W t( ) is the compartment for the concentration of COVID-19 viruses in the environment and

□=______________□ □ □ □(A $_1$+H $_2$+W $_3$+I).

(1.2)

□$_1$D+1

Readers are advised to see Usman et al. (2023), for the description of parameters.

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## 2.0 Analysis of the Model

For analysis, the following system will be considered for Model 1. Since the compartment R appears only in the eighth equation of Model 1. This implies that the remaining equation of (1.1) does not depend on the eight equations, thus it is ignored (See Makinde, 2007; Mpeshe & Nyere, 2021; Yousaf et al, 2022).

dS
__dt = −P □S − K $S_1$ +v $V_2$

dV __dt = v $S_1$ −□(1−b V) − K $V_2$ dE

__=□(S + −(1 b V) )− K $E^3$

dt

dQ __dt =□$_1$E − K $Q_4$ dA __dt =□$_1$E − K $A_5$ dI

__dt =□$_2$A− K $I_6$ dH __dt =□ □$_3$Q+ −$_2$I K $H_7$ dD

__= + +l $A_1$ l $I_2$ l $H_3$ −□D dt $^{dW}$

__ = +□ □□ □I H − W dt

(2.1)

### 2.1 COVID-19 Free Equilibrium

COVID-19 Free Equilibrium (CFE) points are steady-state solutions of the system (2.1) where there is no infection. Hence, in the absence of infection, E Q A I H D $W_0$, $_0$, $_{0\,0}$, , $_0$, $_0$, $_0$ = 0. Then, solve the first and second equation of (2.1) for the non-infected state variables at the equilibrium point to obtain

PK$^2$

(2.2)

$S_0$ =

K $K_{1\,2}$ −v $v_{2\,1}$ and

Pv$^1$ (2.3)

$V_0$ =

K $K_{1\,2}$ −v $v_{2\,1}$

Respectively. Thus, the disease-free equilibrium □$_0$ is given as

□PKPv□

□$_0$(S V E Q A I H D $W_0$, $_0$, $_0$, $_0$, $_{0\,0}$, ,$_0$, $_0$, $_0$)=□ $^{21}$ ,0,0,0,0,0,0,0□

□ K $K_{1\,2}$ −v v K $K_{2\,1}$ $_{1\,2}$ −v $v_{2\,1}$ □

(2.4)

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## 2.2 Computation of Reproduction Number for the Model

Following the next-generation matrix method of computing the reproduction number described by Van den Driessche and Watmough (2002), the matrices for the new infection term and transfer term are respectively obtained as

□o o P□□₁ P A□ ₀P□□₂ P□□₃₀A □ □ K₃ o o o o o□
□o o ₀A o ₀A K₄ o o o o□□
□ o o o o o K₅ o o o□
□o o o o o o −□₂ K₆ o o□□
F =□ o o o o −□₃ o o −□₂ K₇ o□
□o o o o o o □□
□o o o −□ □□ □−
□□o o □ □ □
□ o □ □−□₁
(2.5) o □ □−□
o □,M =□
o □ □ o
□ □ o
□□ □□ o
□ □

The largest eigenvalue of the matrix $FM^{-1}$ is computed with the aid of Maple 18 as

$R_{C1}$ = P A□ □ □ □ □ □□ □□₀( K K₇₄₁(K₆₁ + ₂)+₂₁A + ₃₂A )
(2.6)
□K K K K K₃₄₅₆₇

where,

$A_0$ = ________ev¹ + K₂, $A_1$ = K₄₂₁₂□□□ + K K₅₆₁₃□□ and $A_2$ =□$A_1$ +K K₄₇₂₁₂□□□ (2.7) K K₁₂
−v v₂₁

The next result is valid based on Theorem 2.2 in Van den Driessche and Watmough (2002)

Lemma1: The CFE of (2.1) is locally asymptotically stable if $R_{C1}$ □1 and unstable when $R_{C1}$ □1

### 2.3 Existence of COVID-19 Persistence Equilibrium Point

Let □₊ denote the COVID-19 persistence equilibrium of the model such that its component are obtained from the solution of (2.1) for □ □₊ 0 as

A5 A5 K A3 5
P□□ P□□

Q+ = +1(□ ++e ev1 + K2); A+ = +1(□ ++e ev1 + K2);

K K A3 4 5 K K A3 5 5
P□□□ P□ A

I+ = +1 2(□ ++e ev1 + K2); H+ = +1(□ ++e ev1 + K2)

K K K A3 5 6 5 K K K K K A3 4 5 6 7 5

S+ = P(□ ++e K2); V+ = Pv1 ;
E+ = P□ □ ++( +e ev1 + K2);
D+ = P□+A4 (□ + ++e ev1 K2) and W+ = P□+□A2 (□ + ++e ev1 K2)
(2.8)
□K K K K K A3 4 5 6 7 5□K K K K K A3 4 5 6 7 5

Where A3 = K K4 7 1□(K l6 1 +l2 1□),
A4 = +A3Al1 3, and A5 =□ +□2+e+(eK1 + K2)+ K K1 2 −v v2 1.

Substitute (2.8) into (1.2) with further simplification to get a0□ + □ + =2+ a1 + a2 0 (2.9)
such that

a0 = e□(Px A1 3 +□K K K K K3 4 5 6 7) (2.10)

a1 =□(Px A1 3 +□K K K K K3 4 5 6 7)−P e□□□ □ □ □ □□ □□( K K7 4 1(K6 1 + +2) 2 1A + 3 2A )

(2.11) a2 =e K K K K K KK vv□ 3 4 5 6 7( 1 2 − 2 1)−P e ev K□□( 1 + 2)(□ □ □ □ □ □□K K7 4 1(K6 1 + +2)
e A2 1 + 3 2A )
(2.12)
=e K K K K K KK vv□ 3 4 5 6 7( 1 2 − 2 1)(1−RC1)

It is obvious from (2.10) that a0 □ 0 since all parameters are assumed non-negative and a2 □ 0 from (2.12) if $R_{C1}$ □1. Thus, according to Descarte's rule of sign, (2.9) has a unique positive root (that is COVID-19 persistent equilibrium). Hence, the following result is established.

**Theorem 1.** The model (2.1) has a unique COVID-19 persistent equilibrium if and only if $R_{C1}$ □1

## 2.5. Global Stability of CFE

**Theorem 2.** The COVID-19 free equilibrium of (2.1) is globally asymptotically stable if $R_{C1}$ □1 , and unstable otherwise.

Proof. Following James et al. (2015) and Akinyemi et al. (2016), we consider a Lyapunov function of the form

f =C E1 +C Q C A C I2 + 3 + 4 +C H5 +C W6 (2.13)

where

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$C_1$ = $R_C$ ;

$C_2$ =□□ □□□ □□$_3$( $_3$+ $_2$);$C_4$ =□□ □□□□$_2$( $_3$($_2$+ $K_7$)+□□□ □$_{22}$+ $K_7$)

$S_o$ +e$V_o$ □K $K_{47}$ □K $K_{67}$

(

$C_3$ =□□□□ □□$_{32}$( $_2$ +

$K_7$)+□□□□ □ □ □$_{222}$+ $K_7$( $_2$+ $_{16}$K ));  $C_5$ =□□□□ □□( $_3$+ $_2$)

□K K $K_{467}$ □$K_7$

and $C_6$ ___□□$_3$ (2.14)

□

Differentiate (2.13) with respect to time to have

$$\frac{df}{dt} = C_1\frac{dE}{dt} + C_2\frac{dQ}{dt} + C_3\frac{dA}{dt} + C_4\frac{dI}{dt} + C_5\frac{dH}{dt} + C_6\frac{dW}{dt} \qquad (2.15)$$

Substitute (2.1) into (2.15) appropriately to get

df =C □□□( $_1$A+ +I □$_2$H +n$W_3$ ) K E $S_3$ ( +eV)□□+$C_2$(□$_1$E −K $Q_4$ )+$C_3$(□$_1$E −K $A_5$ )+$C_4$(□$_2$A−K $I_6$ )

$_1$□−

dt □ 1+□$_1$D □

+$C_5$(□ □$_3$Q+ −$_2$IK $H_7$ )−$C_6$(□ □□ □I + H − W) (2.16)

Since ____S +eV □ +$S_o$e$V_o$ = ________P K( $_2$+e$v_1$) then (2.16) becomes

1+□$_1$D K $K_{12}$ −v $v_{21}$

df

__dt □($C_{31}$□ □+ $_{12}$C −C $K_{13}$($S_o$ +e$V_o$))E +($C_{53}$□−C $K_{24}$)Q+(C $S_1$( $_o$ +e$V_o$)□□ □$_1$ +$C_4$ )

($C_1$□($S_o$ +e$V_o$)−c $K_{46}$ +$C_6$□ □+$C_{52}$)I +(C $S_1$( $_o$ +e$V_o$)□□ □□$_2$ +$C_6$ −C $K_{57}$)H − (2.17) (C $S_1$( $_o$

+e$V_o$)□□ □$_3$ −$C_6$ )W

Substitute (2.14) into (2.17) and with further simplifications to obtain df

__dt □□□( $_1$A+ +I □ □$_2$H + $_3$W )($R_{C_1}$ −1) (2.18)

df df

Thus, __□0 if $R_C$ □1with __=0 if and only if A I= = = =H W 0. It follows from Lasalle's dt dt Invariance Principle (LaSalle & Lefschetz, 1961) that □$_o$ is globally asymptotically stable.

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**2.6. Bifurcation Analysis**

To explore the local stability of the COVID-19 persistent equilibrium □₊, bifurcation Analysis will be performed using the centre manifold theory which was described in CastilloChavez and Song, (2004) and Akinyemi et al. (2016). To do this, (2.1) is expressed as

dx₁ = −P □x₁ − K x₁ ₁ +v x₂ ₂ dt

dx₂ = v x₁ ₁ −□ex₂ − K x₂ ₂ dt

dx₃ = □(x₁ +ex₂)− K x₃ ₃ dt

dx₄ = □₁ ₃x − K x₄ ₄ dt

dx₅ = □₁ ₃x − K x₅ ₅ dt

dx₆ = □²A− K x₆ ₆ dt

dx₇ = □ □₃ ₄x + ₂ ₆x − K x₇ ₇ dt dx₈ = l x₁ ₅ +l x₂ ₆ +l x₃ ₇ −□x₈ dt

dx₉ = □ □□ □x₆ + x₇ − x₉ (2.19)

dt by replacing S V E Q A I H D W, , , , , , , , with x x x x x x x x x₁, ₂, ₃, ₄, ₅, ₆, ₇, ₈, ₉ respectively, where

□□( ₁ ₅x + +x₆ □ □₂ ₇x + ₃ ₉x )

□= (2.20) 1+□₁ ₈x

Supposed □ □= * is chosen to be the bifurcation parameter when $R_{C1}$ =1 then (2.6) becomes

□* =□K K K K K₃ ₄ ₅ ₆ ₇

(2.21)

PA₀(□ □ □ □ □□ □□K K₇ ₄ ₁(K₆ ₁ + +₂) ₂ ₁A + ₃ ₂A )

Let U =(u u u u u u u u u₁, ₂, ₃, ₄, ₅, ₆, ₇, ₈, ₉) and W =(w w w w w w w w w₁, ₂, ₃, ₄, ₅, ₆, ₇, ₈, ₉) be the corresponding left and right eigen vectors associated with the zero eigen-value of the Jacobian of (2.19) at □ □= *(denoted by J * ) chosen such that the following conditions are satisfied.

□□=
(i) UJ * = 0

□□=
(ii) J *W = 0

□□=
(iii) UW=1 Thus,

u₁ = 0, u₂ = 0, u₈ = 0,u₃ = u₆(□ □ □ □ □□ □□K K₇ ₄ ₁(K₆ ₁ + ₂)+ ₂ ₁A + ₃ ₂A ),

K K K₃ ₄ ₅(□□□□₃(₂ + K₇)+□□□ □₂ ₂ + K₇)

u K₆ ₆ ₃□ □□ □□□( ₁ + ₃) , u₇ = u K₆ ₆ (□□ □□□₁ + ₃) u₄ =

K₄ (□□□□₃(₂ + K₇)+□□□ □₂ ₂ + K₇)□□□□₃(₂ + K₇)+□□□ □₂ ₂ + K₇

u₅ = u₆ (□□□ □□₃ ₂ (₂ + K₇)+□□□□ □ □ □₂ ₂ ₂ + K₇ ( ₂ + ₁ ₆K )),

K₅(□□□□₃(₂ + K₇)+□□□ □₂ ₂ + K₇)

u₉ = ₆ □□₃ ₆u and u₆ □ 0 free K K (2.22)

(□□□□₃(₂ + K₇)+□□□ □₂ ₂ + K₇)

and

K w ev v₃ ₃( ₂ ₁ + K₂₂) , w₂ =− K v w eK₃ ₁ ₃(₁ + K₂) ,

w₁ =−

(ev₁ + K₂)(K K₁ ₂ −v v₂ ₁) (ev₁ + K₂)(K K₁ ₂ −v v₂ ₁) w₄ =□₁ ₃w K₄

w₅ =□₁ ₃W ,w₆ =□□₁ ₂ ₃W ,w₇ = Aw₁ ₃ ,w₈ = Aw₃ ₃ ,w₉ = A w₂□ ₃ K₅ KK₅ ₆

KKKK₄ ₅ ₆ ₇ □KKKK₄ ₅ ₆ ₇ □KKKK₄ ₅ ₆ ₇

and w₃ □ 0 free. (2.23)

The non-zero second-order partial derivatives at □₀ associated with (2.19) are given by

□₂f₁= □₂f₁=−□□₁, □₂f₁= □₂f₁=−□, □₂f₁= □₂f₁=−□□₂, □ □x x₁ ₅ □ □x x₅ ₁ □ □x x₁ ₆ □ □x x₆ ₁ □ □x x₁ ₇ □ □x x₇ ₁ □₂ f₁ = □₂ f₁ =−□□, □₂ f₂ = □₂ f₂ =−□□₁e, □₂ f₂ = □₂ f₂ =−□e,

3

□ □x x₁ ₉ □ □x x₉ ₁ □ □x x₂ ₅ □ □x x₅ ₂ □ □x x₂ ₆ □ □x x₆ ₂ □₂ f₂ = □₂ f₂ =−□□e, □₂ f₂ = □₂ f₂ =−□□₃e, □₂ f₃ = □₂ f₃ =□□₁

2

□ □x x₂ ₇□ □x x₇ ₂□ □x x₂ ₉ □ □x x₉ ₂□ □x x₁ ₅ □ □x x₅ ₁

□₂ f₃ = □₂ f₃ , □₂ f₃ = □₂ f₃ =□□₂, □₂ f₃ = □₂ f₃ =□□₃,

=□

□ □x x₁ ₆ □ □x x₆ ₁ □ □x x₁ ₇ □ □x x₇ ₁ □ □x x₁ ₉ □ □x x₉ ₁ □₂ f₃ = □₂ f₃ =□□e, □₂ f₃ = □₂ f₃ =□e, □₂ f₃ = □₂ f₃ =□□₂e

1

□ □x x₂ ₅ □ □x x₅ ₂ □ □x x₂ ₆ □ □x x₆ ₂ □ □x x₂ ₇

□ □x x₇ ₂ □₂ f₃ = □₂ f₂ =□□e, □₂ f₁ = □₂ f₁ = □□ □₃PK₂ ₁ ,

3

□x x₂□ ₉□ □x x₉ ₂□ □x x₈ ₉ □ □x x₉ ₈ □(K₂ +v₁)

□₂ f₃ = □₂ f₃ =−□□ □₃P ₁(ev₁ + K₂), □₂ f₁ = □₂ f₁ = □□ □₁PK₂ ₁ ,

□ □x $x_8$ $_9$□ □x $x_9$ $_8$ □$(K_2+v_1)$□ □x $x_8$ $_5$ □ □x $x_5$ $_8$□$(K_2+v_1)$

□$_2$f$_2$ □$_2$f$_2$□ □ □$_1$Pev$_{1\,1}$, □$_2$f$_3$= □$_2$f$_3$= −□□ □$_1$P $_1$(ev$_1$+ K$_2$),

= =

□ □x $x_8$ $_5$ □ □x $x_{58}$□$(K_2+v_1)$ □ □x $x_8$ $_5$ □ □x $x_5$ $_8$ □$(K_2+v_1)$

□$_2$f$_3$= □$_2$f$_3$= −□□P $_1$(ev$_1$+ K$_2$), □$_2$f$_1$= □$_2$f$_1$= □ □PK$_{2\,1}$,

□ □x $x_8$ $_6$□ □x $x_6$ $_8$ □$(K_2+v_1)$□ □x $x_8$ $_6$□ □x $x_6$ $_8$□$(K_2+v_1)$

□$_2$f$_2$= □$_2$f$_2$= □ □Pev$_{1\,1}$, □$_2$f$_3$= □$_2$f$_3$= −□□ □$_2$P $_1$(ev$_1$+ K$_2$) (2.24) □ □x $x_{8\,6}$ □ □x

$x_{6\,8}$□$(K_2+v_1)$ □ □x $x_{8\,7}$□ □x $x_{7\,8}$□$(K_2+v_1)$

□$_2$f$_1$= □$_2$f$_1$= □□ □$_2$P $_1$, □$_2$f$_2$= □$_2$f$_2$= □□ □$_2$Pev$_{1\,1}$

□ □x $x_8$ $_7$ □ □x $x_{78}$□$(K_2+v_1)$ □ □x $x_8$ $_7$ □ □x $x_7$ $_8$□$(K_2+v_1)$

□$_2$f$_2$ □$_2$f$_2$□□ □$_3$Pev$_{1\,1}$

= =

□ □x $x_8$ $_9$ □ □x $x_9$ $_8$□$(K_2+v_1)$

and

□$_2$f$_1$= □$_2$f$_1$= −□$_1$PK$_2$ , □$_2$f$_2$= □$_2$f$_2$= −□$_1$Pev$_1$,

□ □$x_5$□ □ □□ □$x_5(K_2+v_1)$ □ □$x_5$□ □ □□ □$x_5$ $(K_2+v_1)$

□$_2$f$_1$= □$_2$f$_1$= −PK$_2$ , □$_2$f$_2$= □$_2$f$_2$= −Pev$_1$ ,

□ □$x_6$□ □ □□ □$x_6(K_2+v_1)$ □ □$x_6$□ □ □□ □$x_6$ $(K_2+v_1)$

□$_2$f$_3$= □$_2$f$_3$=□$_1$P K( $_2$+ev$_1$), □$_2$f$_3$= □$_2$f$_3$= P K( $_2$+ev$_1$)

□ □$x_5$□ □□ □$x_5$□$(K_2+v_1)$ □ □$x_6$□ □□ □$x_6$□$(K_2+v_1)$

□$_2$f$_1$= □$_2$f$_1$= −□$_2$PK$_2$, □$_2$f$_2$= □$_2$f$_2$= −□$_2$Pev$_1$,

□ □$x_7$□ □ □□ □$x_7(K_2+v_1)$ □ □$x_7$□ □ □□ □$x_7$ $(K_2+v_1)$

□$_2$f$_3$= □$_2$f$_3$□$_1$P K( $_2$+ev$_1$), □$_2$f$_3$= □$_2$f$_3$=□$_2$P K( $_2$+ev$_1$)

=

□ □$x_5$□ □□ □$x_5$□$(K_2+v_1)$ □ □$x_7$□ □□ □$x_7$□$(K_2+v_1)$

□$_2$f$_1$= □$_2$f$_1$= −□$_3$PK$_2$, □$_2$f$_2$= □$_2$f$_2$= −□$_3$Pev$_1$,

□ □$x_9$□ □ □□ □$x_9(K_2+v_1)$ □ □$x_9$□ □ □□ □$x_9$ $(K_2+v_1)$

□$_2$f$_3$= □$_2$f$_3$=□$_3$P K( $_2$+ev$_1$).

□ □$x_9$□ □□ □$x_9$□$(K_2+v_1)$ (2.25)

Using (2.22)-(2.25) such that n=9 to compute a and b as defined in Castillo-Chavez and Song (2004) to get a= $-2\square\ \square\ \square\ \square u\, w^3(\ _{5\,1} + w_{7\,2} + w_{9\,3} + w_6)(Pw_{8\,1}\square\ \square - (ew\ w_2 + {}_1))(ev\ K_1 + {}_2)$ (2.26) $\square(K\ v_2 + {}_1)$

and

b= $u\ w_3(\ _{5\,1}\square\ \square\ \square + w_{7\,2} + w_{9\,3} + w_6)(ev\ K_1 + {}_2)$ (2.27)

$\square(K\ v_2 + {}_1)$

It is obvious to note that b☐ 0 and a☐ 0 since all the parameters values are positive and since $w_1$ ☐ 0 and $w_2$ ☐ 0. Thus, (1.1) exhibits forward bifurcation if $R_{C1}$ ☐1 and the next result is obtained.

**Theorem 3**. The unique COVID-19 persistent equilibrium for model (1.1) is locally asymptotically stable if $R_{C1}$ ☐1 and is close to unity.

## 2.7 Sensitivity Analysis

Sensitivity analysis is carried out using the normalized forward sensitivity index of the reproduction number $R_{C1}$ as discussed in Akinyemi et al. 2023, to investigate the effects of changes in parameter values on the spread of COVID-19. Maple 18 software is used to get the sensitivity indices presented in Table 4.1.

$\square_{\square}^{R_{1C1}} = \frac{\square R_{C1}}{\square\square_1}\frac{\square}{R_{C1}} = -0.01656$; $\square_{\square}^{R_{2C1}} = \frac{\square R_{C1}}{\square\square_2}\frac{\square_2}{R_{C1}} = +0.0782$; $\square_{\square}^{R_{3C1}} = \frac{\square R_{C1}}{\square\square_3}\frac{\square_3}{R_{C1}} = +0.0012$;

$\square_{\square}^{R_{C1}} = \frac{\square R_{C1}}{\square\square}\frac{\square}{R_{C1}} = +0.9288$; $\square_{\square}^{R_{C21}} = \frac{\square R_{C1}}{\square\square_2}\frac{\square_2}{R_{C1}} = -0.0644$; $\square_{\square}^{R_{C31}} = \frac{\square R_{C1}}{\square\square_3}\frac{\square_3}{R_{C1}} = -0.0737$;

$\square_{\square}^{R_{1C1}} = \frac{\square R_{C1}}{\square\square_1}\frac{\square_1}{R_{C1}} = +0.0123$; $\square_{\square}^{R_{2C1}} = \frac{\square R_{C1}}{\square\square_2}\frac{\square_2}{R_{C1}} = +0.0410$; $\square_{\square}^{R_{3C1}} = \frac{\square R_{C1}}{\square\square_3}\frac{\square_3}{R_{C1}} = +0.000004$;

$\square_{\square}^{R_{C1}} = \frac{\square R_{C1}}{\square\square}\frac{\square}{R_{C1}} = 0$; $\square_{v}^{R_{2C1}} = \frac{\square R_{C1}}{\square v_2}\frac{v_2}{R_{C1}} = +0.2235$; $\square_{\square}^{R_{1C1}} = \frac{\square R_{C1}}{\square\square_1}\frac{\square_1}{R_{C1}} = -0.0122$;

$\square_{\square}^{R_{2C1}} = \frac{\square R_{C1}}{\square\square_2}\frac{\square_2}{R_{C1}} = -0.230$; $\square_{\square}^{R_{1C1}} = \frac{\square R_{C1}}{\square\square_1}\frac{\square_1}{R_{C1}} = 0$; $\square_{v}^{R_{1C1}} = \frac{\square R_{C1}}{\square v_1}\frac{v_1}{R_{C1}} = +0.2236$;

$\square_{\square}^{R_{1C1}} = \frac{\square R_{C1}}{\square\square_1}\frac{\square_1}{R_{C1}} = +0.2576$; $\square_{\square}^{R_{C1}} = \frac{\square R_{C1}}{\square\square}\frac{\square}{R_{C1}} = +0.0012$; $\square_{\square}^{R_{C1}} = \frac{\square R_{C1}}{\square\square}\frac{\square}{R_{C1}} = +0.0001$;

$$\square_{\square}^{R_{C1}} = \frac{\square R_{C1}}{\square\square}\frac{\square}{R_{C1}} = -0.0012 \qquad (4.49)$$

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Table 4.1: Sensitivity Analysis Results

| Parameter | Sensitivity index of $R_{C1}$ wrt v$\left(\square_v^{R_{C1}}\right)$ |
|---|---|
| $\square_1$ | +0.0123 |
| $\square_2$ | +0.0410 |
| $\square_3$ | +0.000004 |
| $\square_1$ | −0.0122 |
| $\square_2$ | −0.230 |
| $\square_1$ | −0.1656 |
| $\square_2$ | −0.00644 |
| $\square_3$ | −0.0737 |
| $v_1$ | −0.2236 |
| $v_2$ | +0.2235 |
| $\square_1$ | 0 |
| $\square_1$ | +0.2576 |
| $\square_2$ | +0.0782 |
| $\square_3$ | +0.0012 |
| $\square$ | +0.9288 |
| $\square$ | 0.0012 |
| $\square$ | +0.0001 |
| $\square$ | −0.0012 |
| $\square$ | 0 |

## 3.0 Numerical Simulation

To validate the qualitative results presented in Section 2, we shall make use of the value of parameters and the initial condition in Usman et al. (2023). The effect of varying the initial population of (1.1) for different equilibrium points is shown in Figures 3.1-3.6. In Fig 3.1, the variation of the population of all infected individuals converges to zero when $R_{C1} = 0.6918\square 1$

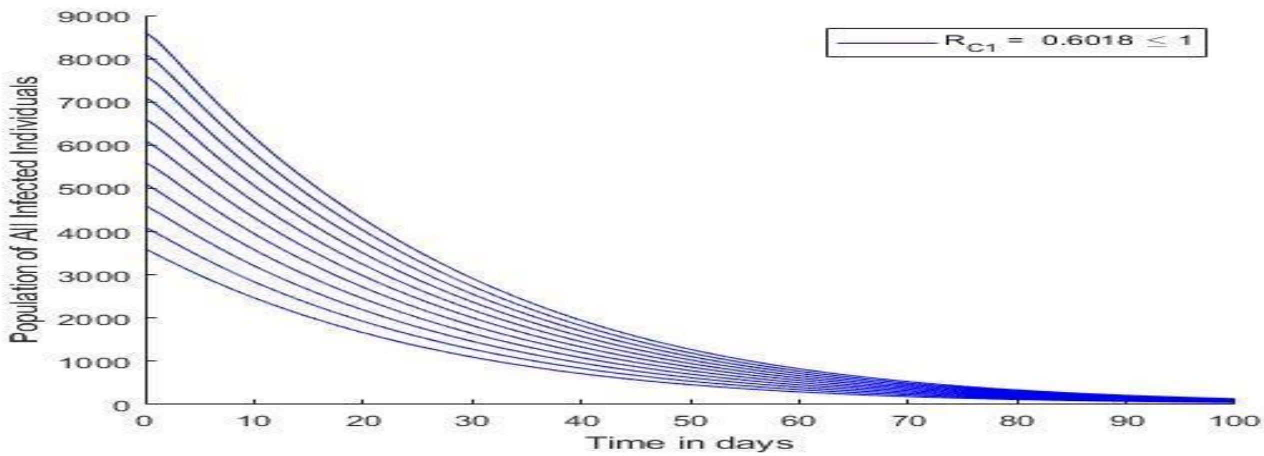


Figure 3.1: Convergence of solution trajectories for all infected humans when $R_{C1} = 0.6918$□1.

Fig 3.2, the variation of the initial population of COVID-19 deceased individuals tends to zero when $R_{C1} = 0.6918$□1.

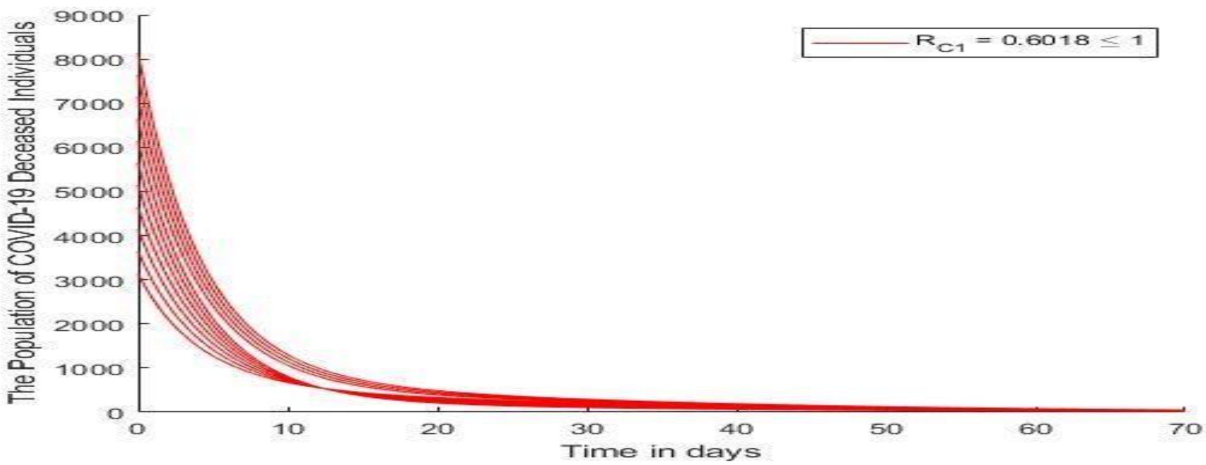


Figure 3.2: Convergence of solution trajectories for COVID-19 deceased individuals when $R_{C1} = 0.6918$□1

The initial concentration of COVID-19 viruses in the environment is varied in Fig. 3.3 and converges to zero at $R_{C1} = 0.6918$□1.

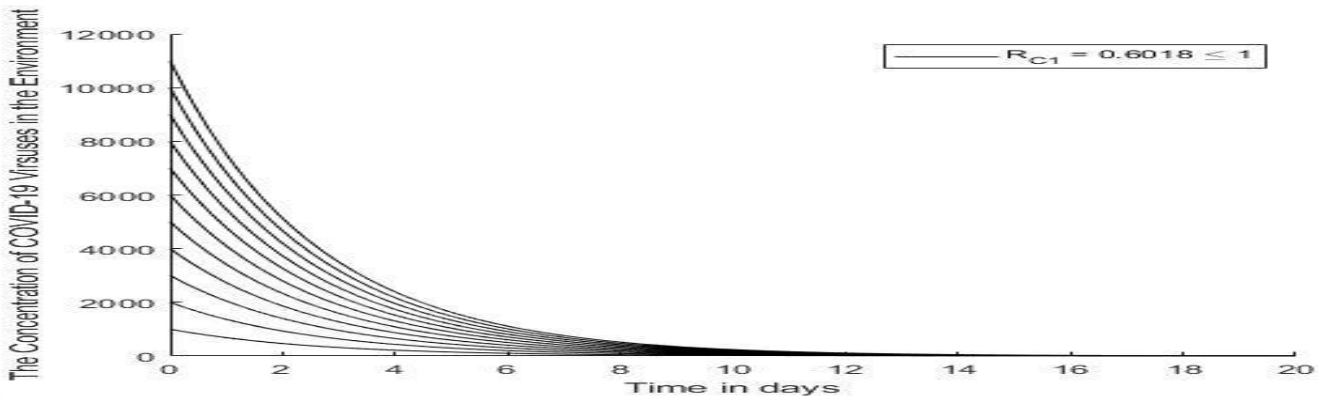


Figure 3.3: Convergence of solution trajectories for COVID-19 Viruses in the environment when $R_{C1}$ = 0.6918□1.

It is important to note that Fig. 3.1-3.3 validates the global stability result for the CFE of (1.1) presented by Theorem 2. Fig.3.4 shows that irrespective of the initial population of all infected individuals, It converges to $E_+ + + + + A_+ Q_+ I_+ H_+$ provided $R_{C1}$ =1.2035□1.

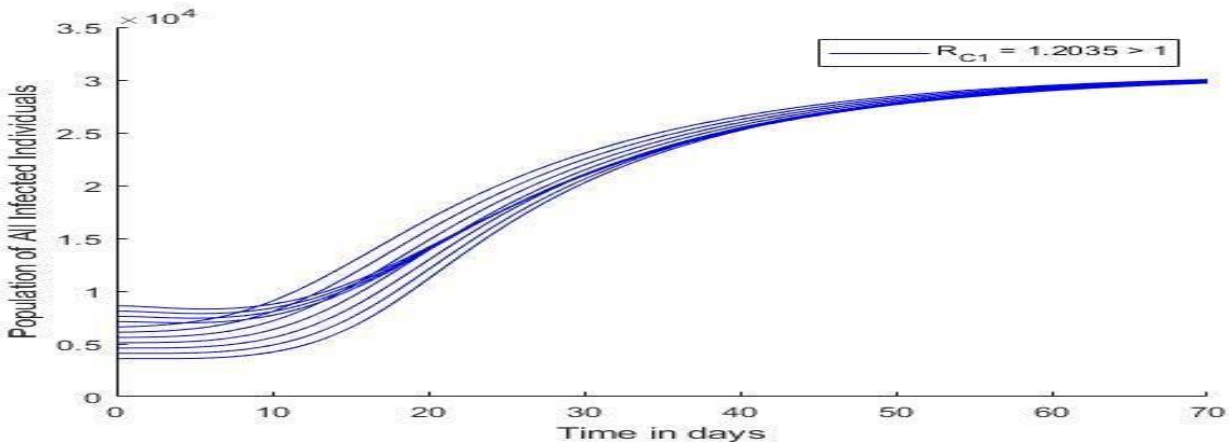


Figure 3.4: Convergence of solution trajectories for all infected humans when $R_{C1}$ =1.2035□1 Similarly, Fig.3.5 shows that irrespective of the initial population of COVID-19 deceased individuals, the population of COVID-19 deceased individuals converges to $D_+$ provided $R_{C1}$ =1.2035□1.

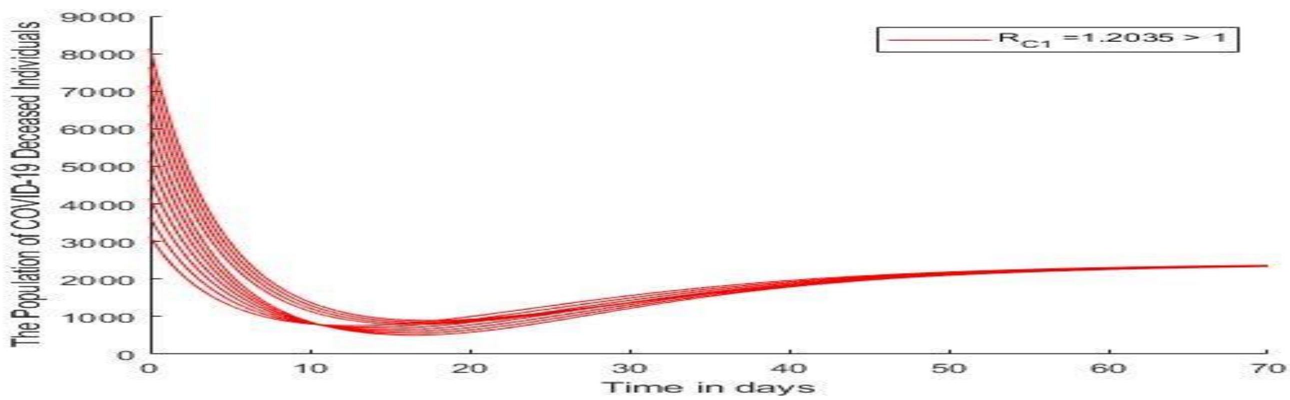


Figure 3.5: Convergence of solution trajectories for COVID-19 deceased individuals when $R_{C1}$ =1.2035□1

Fig. 3.6 shows that provided $R_{C1}$ =1.2035□1, the concentration of COVID-19 viruses in the environment converges to $W_+$ irrespective of the initial concentration.

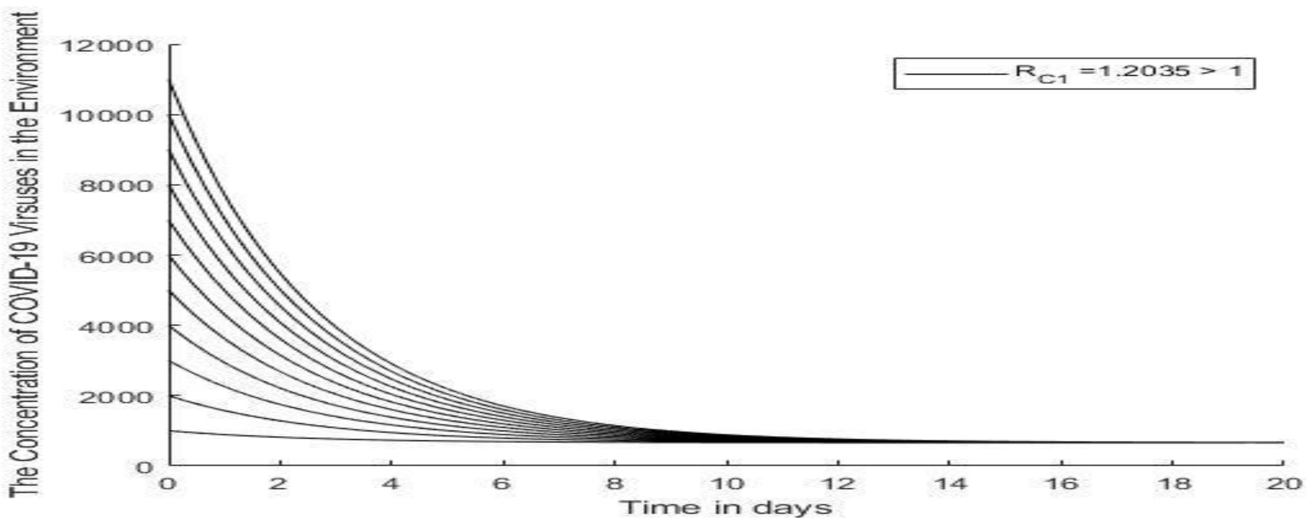


Figure 3.6: Convergence of solution trajectories for COVID-19 Viruses in the Environment when $R_{C1}$ =1.2035□1

The numerical simulation displayed in Fig. 3.4-3.6 establishes that the CPE of (1.1) is globally asymptotically stable when the RC1 = 1.2035 > 1.

### 3.1 Simulation for $\square_1$

The impact of the fear of COVID-19 infection on the transmission dynamics of COVID-19 is, investigated through numerical simulation. This is done by comparing the following five fear levels, namely, no fear ($\square_1$ = 0), low fear level ($\square_1$ = 0.25), moderate fear level ($\square_1$ = 0.5), high fear level ($\square_1$ = 0.75) and very high fear level ($\square_1$ = 1). Fig. 3.7 shows the impact of different levels of fear associated with COVID-19 infection on the population of all infected humans. Fig. 3.7a shows that in the absence of fear ($\square_1$ = 0) associated with COVID-19 infection, there are more COVID-19-infected humans compared to when the fear level is low ($\square_1$ = 0.25). Fig. 3.7b reveals that as the level of fear of COVID-19 infection increases the number of infected human's decreases. This means that as the general public becomes more scared of being infected, there will be a decline in the number of humans with COVID-19 infection. The epidemiological implication of this finding is that the populace will adhere to public health measures to avoid being infected. Thus, this agrees with the findings of Murad et al. (2022), Karlsson et al. (2021) and Recio-Vivas et al. (2020).

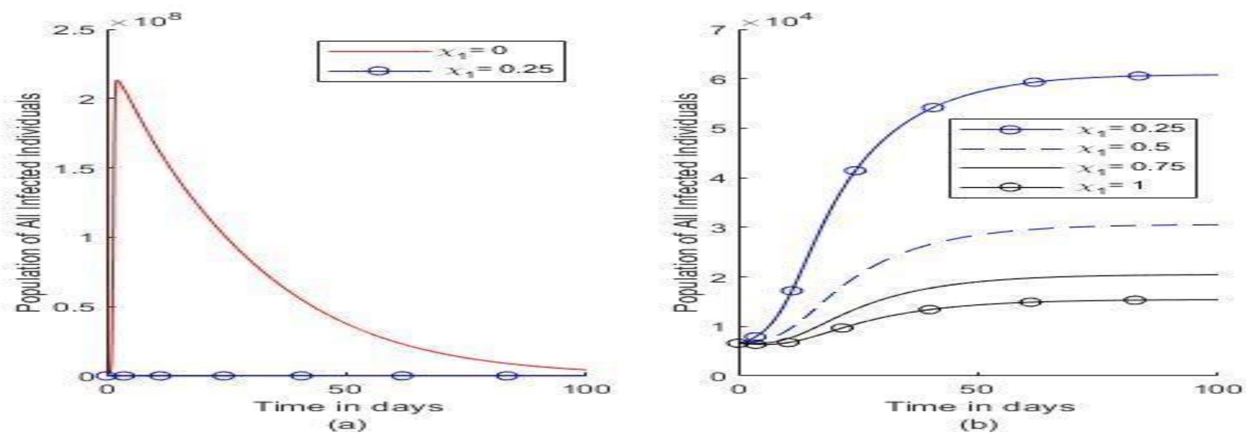


Figure 3.7: Impact of varying $\square_1$ on the Population of All Infected Individuals

The impact of the various levels of fear associated with COVID-19 infection on the population of COVID-19 deceased individuals are shown in Fig. 3.8. Fig. 3.8a shows that there are more COVID-19 induced death cases in the absence of fear ($\square_1$ = 0) as compared to when the fear level is low ($\square_1$ = 0.25). Fig. 3.8b conveys that the decrease in $\square_1$ implies an increase on the number of COVID-19 induced death cases.

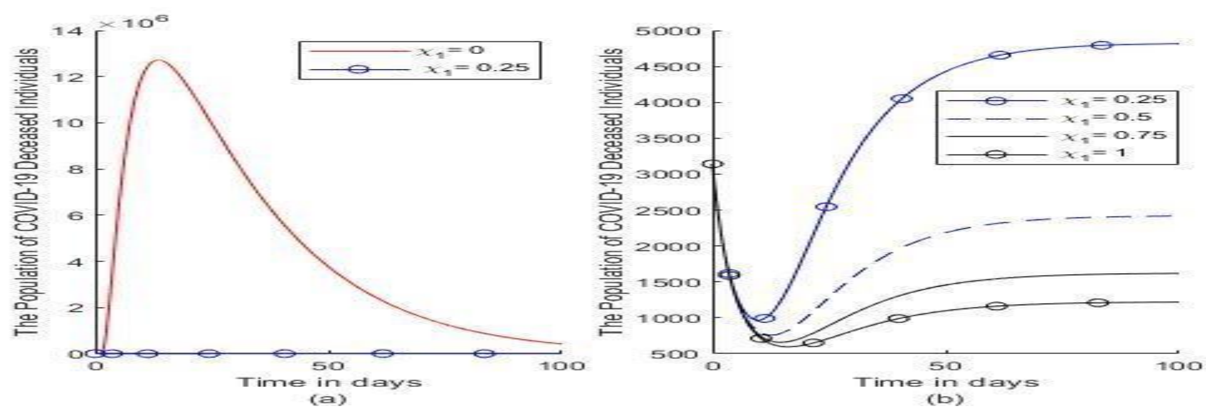


Figure 3.8: Variation of $\square_1$ on the Population of COVID-19 Deceased Individuals

The effect of varying □₁ on the concentration of COVID-19 viruses in the environment is displayed in Fig. 3.9. Similarly, Fig. 3.9a and Fig. 3.9b shows that an increase in □₁ yields decrease in the concentration of COVID-19.

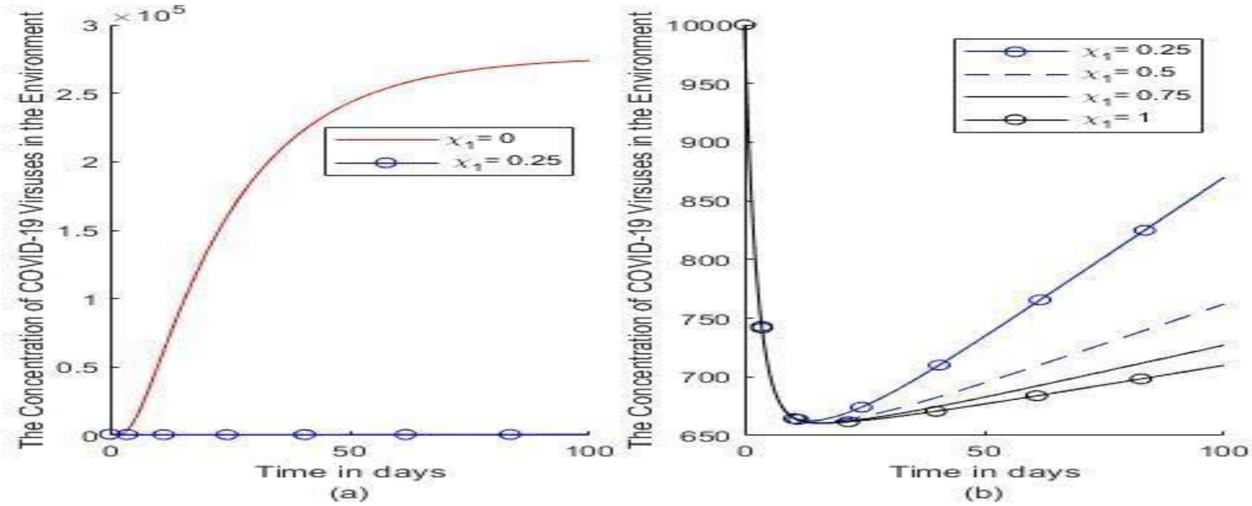


Figure 3.9: Impact of varying □₁ on the Concentration of COVID-19 Viruses

## 4.0 Conclusion

This study presents the qualitative analysis of the COVID-19 mathematical model coupled with the fear of infection in Usman et al., 2023. The two equilibrium points (i.e. the CFE and CPE) of the model were found and the reproduction number (RC1) for the model was computed using the next-generation matrix method. This study further shows that the CPE is locally asymptotically stable when $R_{C1}$ is above unity and the CFE is globally asymptotically stable for less than one. The model is also shown to exhibit forward bifurcation, thus the threshold is sufficient to control the spread of COVID-19. Sensitivity analysis suggests that fear of infection does not influence the $R_{C1}$ but the numerical simulation reveals that as the level of fear of infection increases, the population of all infected humans, COVID-19 deceased individuals and the concentration of the COVID-19 virus increases.

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