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THE ROLE OF SWITCHING COSTS IN SHAPING PERSONALIZED PRICING STRATEGIES

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Abstract: Behavior-based price discrimination is a pricing strategy frequently observed in membership-based services and it has been studied widely in the literature. This paper considers a two-period behavior-based price discrimination model in which there are two distinct types of consumers with different demands, and a common switching cost is incurred for all customers who switch firms in the second period, regardless of customer type. We assume that firms accepting the switching customers bear the switching cost because they aim to attract customers from rival firms. As switching costs increase, competition for higher-demand customers intensifies. Eventually, in the second period, firms stop poaching these customers due to the heavy burden of switching costs. This leads to a situation where only consumers with low demand switch. The equilibrium price in the first period remains positive as long as both types of customers switch in the second period, but it continues to decline and eventually reaches zero after higher-demand customers stop switching. This means that sales promotions such as "first-time free" and "30-day free"offers are justifiable as long as higher-demand customers remain with the same firm in the second period. Once the price in the first period reaches zero, firms' profits increase with the switching cost. We also find that higher switching costs are beneficial from a social welfare perspective, although they are detrimental to consumers.

Keywords Behavior-based price discrimination · Competitive strategy · Poaching · Switching costs · Multiple consumer types

1 Introduction

Behavior-based price discrimination (BBPD) is a common pricing strategy used by firms for their products and services. Under this strategy, firms offer different prices to customers based on their purchase histories. To implement BBPD, firms need to create customer behavior databases and use personalized communication channels to interact with their customers. Recent advances in information and communications technology have made BBPD more feasible. According to Gartner (2022), 80% of major firms have adopted customer relationship management systems, with customer databases forming a crucial part of the modern business model. The goal of firms using BBPD is to establish long-term customer relationships rather than focusing solely on short-term sales. This is believed to maximize the customer lifetime value. As a result, firms might be willing to forgo short-term sales to build enduring customer relationships (e.g., Peppers and Rogers 1993). Some pricing tactics, such as "first-time free" or "30day free" offers, are used widely by many firms to expand their customer base and develop medium- to long-term relationships. These practices are commonly seen in membership-based services such as mobile phones, sports gyms, and water coolers. BBPD has been studied from an economic perspective (e.g., Chen 1997; Fudenberg and Tirole 2000). These studies employ a two-period model, analyzing differences in behavior between the first and second periods. In the second period, firms offer strategic prices to poach customers acquired by the rival firms in the first period. Simultaneously, the firms adjust their prices in the second period to defend their customers from the rival firms. Although BBPD strategies seem effective in securing firms' profits, many studies show that profits are lower in a market where all firms adopt BBPD than in a market without BBPD (i.e., uniform pricing). The interpretation is that if all firms have more pricing options due to technology, their competition intensifies, resulting in lower profits. This is a somewhat ironic outcome for firms that invest in information and communications technology, even if they must do so to survive. However, an alternative explanation is that the lack of positive outcomes could be due to the underutilization of customer information. These models assume that consumers purchase the same amount of goods. In reality, purchase volumes vary from customer to customer; therefore, firms offer better incentives to customers who make larger purchases. For example, the widely used segmentation method—the regency, frequency, monetary (RFM) metric—includes indices representing purchase volumes, i.e., the monetary and frequency aspects (e.g., Kumar and Reinartz 2018). Shin and Sudhir (2010) focus on firms offering incentives to customers who make larger purchases and propose a model in which there are two types of consumers in the market: those who consume less (L-type) and those who consume more (H-type). They prove that if the purchase volume of H-type customers is sufficiently large relative to that of L-type customers, then firms achieve higher profits under BBPD than in the uniform-pricing case. Another factor that determines whether the BBPD works is the switching cost. If switching costs are high, consumers are generally hesitant to switch in the second period; therefore, firms lower prices to gain more market share in the first period. In this paper, we consider a two-period BBPD model where there are two types of consumers with different demands, and a common switching cost is applied to all switching customers, regardless of their purchase quantity. We also assume that firms accepting the switching customers bear the switching costs. Switching costs are naturally incurred by consumers who switch. However, firms offer a variety of compensatory programs to encourage consumers to switch. For example, mobile phones often require changing hardware when switching carriers, and sales promotions frequently offer free hardware to customers who switch carriers. Offering data migration fees and free training for cloud migration or offering free banking accounts are other examples of firms' compensating switching costs. Chen and Sacks (2024) provide more examples of recent U.S. cases. However, even if switching costs are no longer a burden for consumers, consumers will see no benefit from switching if prices remain the same. Therefore, firms must offer discounts on the poaching prices to consumers who switch. In other words, firms offer consumers a double advantage, covering their switching costs and providing discounts.

Our model analyzes such firms' behaviors.

There is another reason for introducing such a model in which firms bear switching costs. Many studies, including Chen (1997), adopt a model in which consumers pay switching costs, and firms, in return, offer price discounts to customers who switch. In this setting, if there is only one type of consumer, or if switching costs are proportional to demand, customers can be compensated by firms for the portion of switching costs that they pay through the firms offering price discounts. However, if the switching costs are the same among consumers in the case of multiple types of consumers, the costs cannot be compensated equally among the different types of customers through discounts in poaching. To analyze the situation where firms bear the common switching cost regardless of consumer type, it is clearer to separate discounts for switching costs from the prices and assume that firms pay the switching costs. The analysis of the effect of switching costs starts with small switching costs s and gradually considers the case of large switching costs. When the switching cost is small, all equilibrium prices in the first and second periods are positive, as in Shin and Sudhir (2010), where no switching cost exists. The price in the first period decreases with s, whereas the price in the second period increases. This phenomenon is consistent with the hypothesis that switching costs force firms to expand their market share in the first period and secure more profits in the second. However, as the rival firm behaves similarly, both firms experience a decrease in their total profits. As for equilibrium prices, the price in the first period is greater than the poaching price in the second period in the case of small switching costs. For larger dealing with symmetric firm situations include (Carroni 2018a; Colombo 2018; Jeong and Maruyama 2018; Chung 2020). Switching costs, the relationship may reverse, but the reversal condition between the prices depends on the discount rate, and is, surprisingly, independent of the composition of the consumer types. In other words, the condition is the same for the case with a single type of consumer.

When switching costs become large, it becomes increasingly difficult and eventually impossible to poach H-type customers from the rival firm in the second period due to the heavy burden of switching costs. In this case, only L-type customers will switch, and the firm should set prices for its existing H-type customers to such a level that the rival firm is not interested in poaching these H-type customers. The price is a limit price, preventing the rival's reentry to poaching activity. By giving up poaching H-type customers, firms can save the large compensation costs that they would otherwise have had to offer switching customers. Nevertheless, we find that total profits remain no increasing with s. After poaching of H-type customers stops, the prices in the first period decrease and eventually reach zero as s increases. Because we do not allow negative prices in our model, the price in the first period must stay at zero thereafter. That is, making a "first-time free" offer becomes optimal for the firms in such a situation. Although the firm's profits in the first period also become zero, the total profits of the firms start increasing with s because the escalation of competition in the first period stops. In real-world businesses, especially membership-based services, we often observe "first-time free" or "30-day free" offers. Our model demonstrates that firms rationally choose such pricing strategies under the existence of large switching costs. We note here that the zero price in the first period is unique to the scheme where firms pay the switching costs. When switching costs are paid by consumers, firms compensate customers for switching costs at different levels depending on whether they are H- or L-type customers. This compensation is provided in the form of discounts. Thus, the total switching cost compensated through discounts in poaching is q times larger for H-type than for L-type customers. Therefore, in most cases, it is possible to poach H-type customers in the second period. As a result, the motivation to capture a larger share in the first period declines, and the price in the first period always remains positive. Some results for the case where consumers pay switching costs are provided in the Appendix. Now, the following question arises: are the behaviors of firms in the presence of switching costs good for society as a whole? We evaluate consumer surplus (CS) and social welfare (SW) in our model, and find that when switching costs is small, CS increases with s because firms are encouraged to compete, whereas SW decreases. In contrast, for a large s, CS decreases to the extent that firms increase their profits, but SW increases. In other words, the existence of switching costs is not harmful for society as a whole although it reduces the CS. Our model belongs to the class of BBPD models where switching costs exist. The early literature on BBPD includes (Chen 1997; Villas-Boas 1999, 2004; Fudenberg and Tirole 2000; Shaffer and Zhang 2000; Taylor 2003). Since the early 2000 s, BBPD has been the subject of numerous studies in various settings. The seminal research on switching costs is by Klemperer (1987a, 1987b), with comprehensive surveys provided by Klemperer (1995) and Farrell and Klemperer (2007). These studies show that consumer switching costs tend to reduce market competitiveness. Chen (1997) develops a model in an undifferentiated market, the first to combine BBPD and switching costs, and concludes that competition eases in the second period, as in Klemperer (1987a, 1987b), whereas in the first period, the competition intensifies as switching costs increase, resulting in prices below marginal costs. In our context, this implies negative prices in the first period. We show that in a differentiated market with multiple consumer types, the prices in the first period stay positive as long as switching by both L- and H-type customers in the second period occurs in equilibrium. However, the prices can be zero, i.e., "first-time free," when switching of H-type customers stops. We explicitly identify the condition under which zero prices occur. According to Umezawa and Yamakawa (2024), if there is only one type of consumer in a differentiated market, firms can maintain positive prices even in the presence of switching costs. Other studies that consider switching costs with BBPD include (Taylor 2003; Gehrig and Stenbacka 2004; Chen 2008; Esteves 2010; Jeong and Maruyama 2018, 2009; Colombo 2015; Umezawa 2022). However, in all these studies, the consumers bear the switching costs. In other words, consumers suffer negative utility when they switch brands. In contrast, in this paper, we assume that firms pay the consumers' switching costs. The remainder of this paper is organized as follows. In Sect. 2, we introduce a model of BBPD with switching costs. In Sect. 3, we find the equilibrium prices and profits not only for small switching costs, but also for high switching costs. Based on the equilibrium analysis, numerical examples are given for typical parameters. Moreover, we evaluate the equilibrium solutions from the perspectives of the CS and SW. In Sect. 4, we consider partial uniform pricing as a benchmark and compare it with BBPD. Section 5 concludes our paper. The proofs are provided in the Appendix.

2 The model

This paper considers a two-period duopoly model consisting of two firms, A and B. We assume that both firms are located at the endpoints on the line segment [0, 1], which represents the product characteristic space. Let A be located at 0 and B at 1. We assume that the locations of the firms do not change over the two periods. Both firms sell an identical nondurable good. Let v be the value per unit of goods that a consumer retains, which is assumed to be sufficiently large that all consumers purchase the goods from one of the firms in equilibrium. In addition, the production cost of the goods is assumed to be zero. We assume that there are two types of consumers in the market: H-type consumers, who purchase q (>1) units of the goods in each period, and L-type consumers, who purchase one unit, where q is exogenously given. In other words, H-type consumers are prospective customers who produce high customer lifetime value from the perspective of customer relationship management. For simplicity, we write the quantity demanded by k-type customers ($k \in \{L,H\}$) as Q^k , where $Q^L = 1$ and $Q^H = q$. The consumers have different brand preferences, which remain unchanged over the two periods and are uniformly distributed on [0, 1] with density one. We assume that the proportions of H-type and L-type customers are α and 1− , respectively, where $0<\alpha<1$. Consumers incur disutility from purchasing a product variety that differs from their ideal variety. Thus, when the price of the product offered by firm $i \in \{A,B\}$ is p^i , the per-period utility of a k-type consumer at $x \in [0,1]$ is given by $Q^k(v-p^A)-x$ if the consumer purchases from firm A and $Q^k(v-p^B)-(1-x)$ from firm B. We assume that the unit transportation cost is 1. The firms compete on prices for two periods. Firms attempt to incorporate information on customer demand and purchase histories into their pricing to acquire and retain consumers who consume more. However, because no consumer is a customer of either firm at the beginning of the first period, the firms do not have any information on consumer types. Thus, in the first period, each firm $i \in \{A,B\}$ sets a uniform price $p^i{}_1$. In the second period, firms engage in BBPD as follows. At the beginning of the second period, the firms know their own customers' types and thus can set their prices based on these customer types. Let $p^{ik}{}_2$ be the price offered by firm i to its existing customers of k-type ($k \in \{L,H\}$) in the second period. However, neither firm has information on the types of customers of the rival firm; therefore, firm i sets a price $p^{iR}{}_2$, called a poaching price, for both the L- and H-type customers who purchased the rival's goods in the first period. That is, the prices offered to attract the rival's customers cannot be differentiated by consumer type. We assume that all prices in the two periods are nonnegative. We assume that the discount rate δ $(0<\delta\leq 1)$ is common among consumers and firms. Our model is based on Shin and Sudhir (2010). They assume that consumer preferences change probabilistically between the first and second periods, whereas, to focus on switching costs, we assume that preferences do not change. That is, our model corresponds to their model with switching costs and the probability $\beta=0$. Switching costs are incurred for customers that a firm poaches from the rival in the second period. In this paper, we assume that the firm poaching the customers ≥ compensates them for the switching costs (s 0). This situation corresponds to incentives, such as cash-back offers, offered to customers who switch from a rival firm, often seen in membership-based services, such as mobile phones or Internet services. We note that switching costs are assumed to be the same regardless of

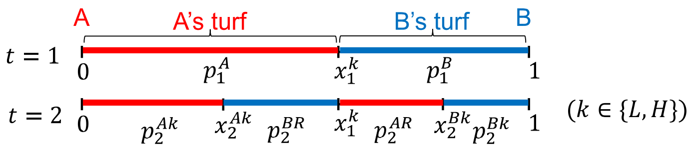

Fig. 1 Two-period Hotelling market with two types of consumers

consumption levels. This assumption corresponds with real-world cases, such as the mobile phone industry in which handsets are offered free of charge upon switching, regardless of the amount used in the previous period. The timing of the game in each period is as follows. In the first period, firms *A* and *B* simultaneously set prices $p^A{}_1$ and $p^B{}_1$. They are aware that consumers will decide from which firm to purchase goods by considering the prices that the firms will set in the second period. In the second period, each firm $i \in \{A,B\}$ sets prices p_2^{ik} $(k \in \{L, H\})$ for its existing *k*-type customers and a poaching price $p^{iR}{}_2$ for its rival's customers. Consumers decide from which firm to purchase the goods. Having knowledge of this process, the firms determine prices to maximize their profits throughout the entire period.

3 Analysis

3.1 Equilibria under small switching costs

The subgame perfect equilibrium in pure strategies is used to analyze the game. It can be derived by backward induction as usual. First, consider the second period of the game. We have four segments for each market of *k*-type customers ($k \in \{L,H\}$) in the second period according to the combinations of firms from which a consumer elects to buy the goods in the first and second periods (see Fig. 1).

Let $x_2{}^{ik}$ $(i \in \{A,B\}$, $k \in \{L,H\})$ denote the cutoff location of *k*-type consumers who bought from firm *i* in the first period and who are indifferent between buying good *A* or good *B* in the second period. The locations $x_2{}^{Ak}$ and $x_2{}^{Bk}$ can be determined for $k \in \{L,H\}$ as follows:

Qk $(v - p_2^{Ak}) - x_2^{Ak} = Q^k(v - p_2^{BR}) - (1 - x_2^{Ak})$, Qk $(v - p_2^{AR}) - x_2^{Bk} = Q^k(v - p_2^{Bk}) - (1 - x_2^{Bk})$.

Thus,

($Akp^{BR} - p2^{Ak})Q^k + 1$ Bk $(p2^{Bk} - p2^{AR})Q^k + 1$ (1)

2 x =and x = . 22 2 2

Let $x_1{}^k$ be the cutoff of *k*-type consumers who are indifferent between buying from *A* and *B* in the first period. Firms maximize their profits by observing consumers' behaviors in the second period, given the cutoff x_1^k $(k \in \{L, H\})$ in the first period. On *A*'s turf, firm *A* attempts to defend its own customers, whereas firm *B* attempts to poach customers from firm *A*. The following problems are solved on *A*'s turf in the second period by firms *A* and *B*, respectively:

$_p\max AL2$ $_pAH2$ $\pi_2AA = p_2AH\alpha qx_2AH + p_2AL(1-\alpha)x_2AL,$ (2)

,

$\max^p BR$ $\pi 2BR = p2BR(\alpha q(x1H - x2AH) + (1-\alpha)(x1L - x2AL))$

2 (3)
$-s[\alpha(x1H - x2AH)+(1-\alpha)(x1L - x2AL)]$.
By solving the first-order conditions (FOCs), we obtain the following optimal prices:
$pAL2 = \frac{((-4x_1^L + q(4x_1^H + 2s - 1) - 2s + 3q^2 - 2)\alpha + 4x_1^L + 2s + 2)}{6((q^2-1)\alpha + 1)}$,
$p^{AH}{}_2 = \frac{((q(-4x_1^L - 2s + 1) + q^2(4x_1^H + 2s + 2) - 3)\alpha + q(4x_1^L + 2s - 1) + 3)}{6q((q^2-1)\alpha + 1)}$, and $pBR_2 =$
$\frac{((-4x_1^L + q(4x_1^H + 2s - 1) - 2s + 1)\alpha + 4x_1^L + 2s - 1)}{3((q^2-1)\alpha + 1)}$.

(4)
On B's turf, the roles of firms A and B are reversed. The profit- maximization problems of firms A and B on B's turf in the second period are, respectively:
$\max pAR\ \pi 2AR = p2AR(\alpha q(x2BH - x1H)+(1-\alpha)(x2BL - x1L))$

2
$BB - BHs[\alpha(x2BH - BHx1H)+(BL1-\alpha)(x2BL - x1LBL)]$, (5) $\max BL2\ pBH\ \pi_2 = p_2\ \alpha q(1 - x_2)+p_2(1-\alpha)(1 - x_2)$.
p , 2
By solving the FOCs, we obtain the following optimal prices on B's turf:
$p_2AR = \frac{-((-4x_1^L + q(4x_1^H - 2s - 3) + 2s + 3)\alpha + 4x_1^L - 2s - 3)}{3((q^2-1)\alpha + 1)}$,
$p_2BL = \frac{((4x_1^L + q(-4x_1^H + 2s + 3) - 2s + 3q^2 - 6)\alpha - 4x_1^L + 2s + 6)}{6((q^2-1)\alpha + 1)}$, and
$p_2BH = \frac{-((q(-4x_1^L + 2s + 3) + q^2(4x_1^H - 2s - 6) + 3)\alpha + q(4x_1^L - 2s - 3) - 3)}{6q((q^2-1)\alpha + 1)}$.
(6)
We note that $\pi_2{}^{AA}$ and $\pi_2{}^{BB}$ are the profits that firms A and B derive from their turfs, respectively, whereas $\pi_2{}^{AR}$ and $\pi_2{}^{BR}$ are the respective profits that each firm derives by poaching customers from their rival. As in Eqs. (3) and (5), switching costs are assumed to be covered by the firms. In addition, let $\pi_2{}^i \equiv \pi_2^{ii} + \pi_2^{iR}\ (i \in \{A, B\})$. -Now, we consider how consumers choose firms in the first period. Consumers anticipate that if they buy from A in the first period, firm A will offer a loyalty price to its existing customers to retain them in the second period. In addition, the consumers consider that firm B will provide an attractive price to A's customers to encourage them to switch from firm A to firm B. The consumer of k-type ($k \in \{L,H\}$) at $x_1{}^k$ is indifferent between buying from firm A in the first period and then buying from firm B in the second, or buying from firm B in the first period and then buying from firm A in the second period (see Fig. 1). Therefore, the following equality holds for $k \in \{L,H\}$:
$Q^k(v - p_1{}^A) - x_1{}^k + \delta(Q^k(v - p^{BR}{}_2) - (1 - x_1{}^k)) = Q^k(v - p_1{}^B) - (1 - x_1{}^k) + \delta(Q^k(v - p^{AR}{}_2) - x_1{}^k)$.
(7)

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Thus, $\frac{1}{3+?-3Q^k(p_1^A-p_1^B)}$ $-\frac{}{2}+\frac{(p_1^A-p_1^B)-?(p_2^{AR}-p_2^{BR})}{2(-1+?)}$ $x^k = {}^1 Q^k$. By substituting Eqs. (4) and (6), we have
$x1k = 2\delta$.

6+

The respective profit functions of firms A and B in the first period are given as follows:
$\pi 1^A = p^A{}_1(\alpha q x_1{}^H + (1-\alpha)x_1{}^L)$, $\pi^B = p^B{}_1(\alpha q(1-x_1{}^H)+(1-\alpha)(1-x_1{}^L))$.

1

Each firm determines the optimal price in the first period, taking into account the profit in the second period, as follows:
$\max p A1\ \pi A = {}^{\pi}{}_1 A + ({}^{\pi}{}_2 AA + {}^{\pi}{}_2 AR), \max p\ B1\ \pi B = {}^{\pi}{}_1 B + \delta({}^{\pi}{}_2 BB + {}^{\pi}{}_2 BR)$. (8)
Hereafter, we express equilibrium prices, profits, and so on, using the hat symbol. We introduce a set of conditions of α and q guaranteeing the existence of an equilibrium where switching occurs among both L- and H-type customers in the second period. **Definition 1** Condition (C) is the set of α and q satisfying
$q \leq 3$
or
$q-3$
$q > 3$ and $\alpha \geq \frac{}{(q-1)(2q+3)}$. (9)
We write the complement set of Condition (C) as Condition (C) , i.e., $q>3$ and
$\alpha < \frac{q-3}{(q-1)(2q+3)}$.
Condition (C) is independent of s and, as shown by the proof in the Appendix, it corresponds to the condition for which an equilibrium exists in the case where $s = 0$. We need an extra condition for a positive s. Then, we have the following proposition:
Proposition 1≤ ≤ *Assume Condition* (C) *holds. Then, for the switching cost s satisfying*
0 s s_1 , *where*
$s_1 \equiv \frac{(q-1)(2q+3)\,\alpha - q + 3}{2q((q-1)\alpha + 1)}$, (10)
we have the prices, the cutoffs, and the total profits of the firms in equilibrium as follows:
$\hat{p}A1 = \hat{p}B1 = \frac{(\alpha(q-1)+1)(\delta - 2s\delta + 3)}{3((q^2-1)\alpha+1)}$, (11)
$\hat{p}AL_2 = \hat{p}BL_2 = \frac{(q-1)(2s+3q+4)\alpha + 2s + 4}{6((q^2-1)\alpha+1)}$, (12)
$\hat{p}AH_2 = \hat{p}BH_2 = \frac{(q-1)(2qs+4q+3)\alpha + 2qs + q + 3}{6q((q^2-1)\alpha+1)}$, (13)
$\hat{p}AR_2 = \hat{p}_2^{BR} = \frac{(q-1)(2s+1)\alpha + 2s + 1}{3((q^2-1)\alpha+1)}$,
$\hat{x}_1^L = \hat{x}_1^H = 1/2$,
$\hat{x}^{AL}_2 = \frac{(q-1)(2s+3q+4)\alpha + 2s + 4}{12((q2-1)+1)}$,

AH (q− 1)(2qs+ 4q+ 3)α + 2qs+q+ 3
$\hat{x}2$ = 12((q2 − 1) + 1) ,
BL (q− 1)(−2s+ 9q+ 8)α − 2s+ 8
$\hat{x}2$ = 12((q2 − 1) + 1) ,
BH −((q− 1)(2qs− 8q− 9)α + 2qs+q− 9)

$$\hat{x}_2 = \frac{}{12((q^2-1)\alpha+1)},$$

$$\hat{\pi}^A = \hat{\pi}^B = \frac{}{72((q^2-1)\alpha+1)}\left[36(q\alpha-\alpha+1) + \left\{(q-1)\ (8s\ + 2s + 23)\right.\right.$$
$$\left.\left. +(q-1)(16s^2 - 18qs - 14s + 9q + 55)\alpha + 8(s^2 - 2s + 4)\right\}\delta\right].$$

1 2 2 2 α2
(14)
When switching costs are small, the equilibria can be derived from the FOC for profit maximization. However, under high switching costs, the prices in Proposition 1 are not equilibrium prices. Proposition 1 identifies the range of switching costs where firms poach both *L*- and *H*-type rival-firm customers. The proof confirms that when $s > s_1$, firms do not poach the *H*-type customers of their rival firm (see the Appendix). Moreover, from Proposition 1, we can establish the following result.

Corollary 1 *No poaching of H-type customers occurs if* (*i*) *Condition* (C) *and* $s \geq s_1$ *hold, or* (*ii*) *Condition* (C) *holds.*

Note that in case (ii), Condition 1 (C) does not depend on ≥ s. In fact, under Condi1 tion (C) , s becomes negative; therefore, any s 0 satisfies $s > s$, meaning that no switching occurs among *H*- type customers. Although prices and cutoff points in Proposition 1 appear complex, simple relationships can be found when comparing customer types. The following results are derived directly from the results of Proposition 1.

Proposition 2 (Relative Relationships Between Customer Types≤ 1) *Assume that Condition* (C) *and* $s\ s$ *hold, i.e., switching occurs among both L- and H-type customers in the second period.*

(a) *The prices in the first period decrease with s, whereas the prices in the second period increase. Moreover, the following relationships hold among the equilib≠ rium prices for i,j* ∈{*A*,*B*} *such that i j* :

1 $\partial\hat{p}i1$ $\partial\hat{p}iL2$ $\partial\hat{p}iH2$ 1 $\partial\hat{p}iR2$ 1 $\partial\hat{p}jR2$ ≥
− = = =2 = 0.

2δ ∂s ∂s ∂s ∂s 2 ∂s

(b) *The following relationship holds among equilibrium prices:* $\hat{p}_2^{iR} \leq \hat{p}_2^{iH} \leq \hat{p}^{iL}_2$ *for* $i \in \{A,B\}$. *Moreover, the difference between the prices for L- and H-type customers among each firm's existing customers in the second period is always constant and independent of the switching cost s. Specifically, the following holds for i* ∈{*A*,*B*} :

$$\hat{p}_2^{iL} - \hat{p}_2^{iH} = \frac{q-1}{2q}. \quad (15)$$

(c) *In the second period, there is always more switching by L-type customers than among H-type customers. Specifically, we have the relationships* $\hat{x}_2^{AH} - \hat{x}_2^{AL} > 0$ *and* $\hat{x}_2^{BL} - \hat{x}_2^{BH} > 0$. *Moreover, the difference in switching volumes widens with s*. The higher the switching cost, the lower is the risk of customer defection, so firms will raise the price in the second period (see (a)). Because this behavior is the same for both types of customers, the relative price difference remains the same (see (b)). If a firm takes a customer from a rival firm, it incurs switching costs, so poaching prices rise faster than prices for existing customers to compensate for these costs. As a result, competition softens in the second period as switching costs rise. Hence, firms lower prices in an attempt to gain more market share in the first period and capture profits in the second period (see (a)). The differences between the prices for the existing customers and those offered to attract the rivals' customers are always larger for *L*-type customers than for *H*-type customers, which means that *H*-type customers receive a volume discount. Moreover, the benefit of the discount is greater as *q* increases (see (b)). As a result, the switching consumer segment is larger among *L*-type consumers than among *H*-type customers. In other words, firms can reduce the outflow of *H*-type customers by conducting BBPD. The difference in defections widens as switching costs increase (see (c)). This can be easily verified because

$$\hat{x}_2^{AH} - \hat{x}_2^{AL} = \hat{x}_2^{BL} - \hat{x}_2^{BH} = \frac{(2s+1)(q-1)((q-1)\alpha+1)}{12((q^2-1)\alpha+1)}.$$

Although firms want to acquire more *H*-type customers, this proposition shows that *H*-type customers will gradually stop switching in the second period as switching costs *s* increase, even if the firms bear the switching costs. For example, if smartphones become more sophisticated and expensive, or if the compatibility of cloud services is low, and the migration cost becomes high, firms will be unable to offer a price low enough to make *H*-type customers switch due to the burden of *s*. Ironically, *L*-type customers who cannot get enough volume discounts in the second period continue to switch relatively more than *H*-type customers. This phenomenon can also be confirmed by focusing on the prices in the proposition. To encourage *H*-type customers to switch in the second period as *s* increases, firms have to set the poaching price lower than the price for *H*-type customers, i.e., $\hat{p}_2^{jR} < \hat{p}_2^{iH}$. The price $\hat{p}_2^{jR}$ approaches $\hat{p}_2^{iH}$ as *s* increases from zero. Eventually, we have $\hat{p}_2^{jR} = \hat{p}_2^{iH}$ at $s = s_1$, and the switching of *H*-type customers ceases. However, because the firms care more about *H*-type customers and exert more effort to retain them than they exert for *L*-type customers, firms maintain a lower price for *H*-type customers than the price for *L*-type customers, resulting in the continuation of switching among *L*- type customers. Suppose that the switching costs become high, and the firms know it will be difficult to encourage *H*-type customers to switch in the second period. In that case, firms decrease the price in the first period, and may eventually set the price to zero in the first period to attract more *H*-type customers. Especially when the discount rate δ is large, the price in the first period declines rapidly, as in (a). However, if some *H*-type customers switch in the second period, firms will maintain positive prices in the first period.

Proposition 3 *Assume that Condition* (C) *and* $s \le s_1$ *hold, i.e., switching occurs among both L- and H-type customers in the second period. Then, the price in the first period is always positive.*
This is proved as a part of the proof of Proposition 1 in the Appendix.
In Sect. 3.2, we show that for large switching costs, the price in the first period can be zero. In other words, it indicates that first-time free pricing is rational only when *H*- type customers cannot be poached in the second period.
The relationships between the prices in the first period and the prices in the second period are not simple. For example, the magnitude relationship between $\hat{p}^{i}_1$ and $\hat{p}^{iR}_2$ reverses, depending on the size of *s*.
Proposition 4 *Assume Condition* (C) *and* $s \le s_1$ *hold. We have* $\hat{p}^{i}_1 \ge \hat{p}_2^{iR}$ *if* $s \le \frac{2+\delta}{2(1+\delta)}$ *and* $\hat{p}^{i}_1 < \hat{p}^{iR}_2$ *otherwise.*
See the Appendix for the proof. Here, the threshold depends only on the discount rate , and not on *q* or α. Therefore, this condition holds even for the case where only a single type of consumer exists. This is presumably because both the price in the first period and the poaching price are offered equally to both types of consumers. However, another threshold s_1 that appears in Proposition 1 does not depend on . In other words, ≤ these two thresholds are independent. We note that $\frac{3}{4} \le \frac{2+\delta}{2(1+\delta)} < 1$ because $0<\delta 1$.
Before examining the case where $s \ge s_1$, let us examine the change in the firm's total profits due to switching costs. As discussed above, switching costs intensify the firstperiod competition to acquire customers, which causes a decline in the firms' profits.
Proposition 5 *Assume Condition* (C) *and* $s \le s_1$. *The total profit of each firm in equilibrium decreases with s.*

3.2 Equilibria under large switching costs

In the discussion thus far, we have observed what happens when switching costs increase from zero to s_1. In this subsection, we consider the case where the switching costs are beyond the upper bound. Again, we solve the game by backward induction.≥ 1
Consider the second period of the game. When *s s* , no switching occurs among *H*-type customers (see Fig. 2). However, as switching continues to occur for *L*-type customers, we have the same cutoffs as Eq. (1) for *k*=*L* , which are:

1 1

$x2AL = (1 - pAL2 + pBR2)$ and $x2BL = (1 - pAR2 + pBL2)$.

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2 2

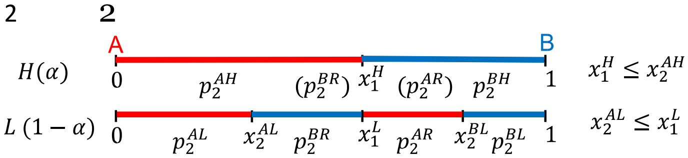

Fig. 2 Market structure in the second period when switching costs are large

Each firm $i \in \{A,B\}$ wants to increase the price in the second period for its exist≠ ing H-type customers because the rival firm ≥j (1$j \in \{A,B\}$;j i) has given up poaching the H-type customers of firm j when s s . However, if firm i's price for existing customers is too high, firm j will resume poaching firm i's H-type customers. Hence, firm i must determine the maximum price that will prevent the rival firm j from reentering i's turf: that is, a limit price. We estimate the limit price below. Consider firm A's limit pricing. Suppose that switching continues to occur for H-type customers, as in Sect. 3.1. In this case, $x_2^{AH} \leq x_1^H$ holds, and we have the same cutoff as Eq. (1) for $k=H$, which is

$$x_2^{AH} = \frac{1}{2}\left(1 - qp_2^{AH} + qp_2^{BR}\right). \qquad (16)$$

From Eq. (14), $p^{BR}2$ increases with s. Therefore, from Eq. (16), x_2^{AH} eventually reaches x_1^H as s increases, i.e., $x_2^{AH} = x_1^H$, which is the situation where firm B gives up poaching from A's turf. Because firm A tries to prevent firm B from poaching its H-type customers, it needs to set the maximum $p^{AH}{}_2$ under the condition that $x_1^H \leq x_2^{AH} = \frac{1}{2}\left(1 - qp_2^{AH} + qp_2^{BR}\right)$. Therefore, firm A should choose a price

$$p^{AH}2 = p_2^{BR} + \frac{1}{q}\left(1 - 2x_1^H\right) \qquad (17)$$

so that $x_1^H = x_2^{AH}$ holds.

Similarly, for firm B's limit price, it follows from $x_1^H = x_2^{BH}$ that

$$p^{BH}2 = p_2^{AR} + \frac{1}{q}\left(-1 + 2x_1^H\right). \qquad (18)$$

In the second period, firms A and B maximize their profits on A's turf with $p^{AL}{}_2$ and $p^{BR}{}_2$, respectively, as follows:

$$\max_{p^{AL}2} \pi_2^{AA} = p^{AH}{}_2\, \alpha q x_2^{AH} + p_2^{AL}(1 - \alpha)x_2^{AL}, \qquad (19)$$

$$\max_{p_2^{BR}} \pi_2^{BR} = p^{BR}2\,(1 - \alpha)(x_1^L - x_2^{AL}) - s(1 - \alpha)(x_1^L - x_2^{AL}). \qquad (20)$$

Note that when $s \leq s_1$, firm A determines ≥ 1 $pAH2^{AH}{}_2$ in addition to $p^{AL}{}_2$ in its profit-maximization problem (2). For a larger s s , p is obtained in Eq. (17), and the maximization problem (19) of firm A is not optimized with respect to $p^{AH}{}_2$. Now, through the FOCs of the problems (19) and (20) , we obtain

$p_2^{AL} = \frac{1 + s + 2x_1^L}{2}$ and $p^{BR}{}_2 = \frac{-1 + 2s + 4x^L}{3}$.

It follows from Eq. (17) that

$$p_2^{AH} = \frac{3 - 6x_1^H + q(-1 + 2s + 4x_1^L)}{3q}.$$

Similarly, we can formulate the profit optimization problems on B's turf as follows:

$$\max_{p_2^{BL}} \pi_2^{BB} = p^{BH}2\, \alpha q(1 - x_2^{BH}) + p_2^{BL}(1 - \alpha)(1 - x_2^{BL}),$$

$$\max_{p_2^{AR}} \pi_2^{AR} = p^{AR}2\,(1 - \alpha)(x_2^{BL} - x_1^L) - s(1 - \alpha)(x_2^{BL} - x_1^L).$$

From the FOCs of these problems and Eq. (18) , we obtain
BL 3 + s − 2x_1^L AR 3 + 2s − 4x_1^L BH −3 + 6x_1^H + q(3 + 2s − 4x_1^L) p_2 = 3 , p_2 = 3 , and p_2 = 3q .

Consider the first period of the game. We want to know the cutoff x_1^k of the first period in each $k \in \{L,H\}$. As H-type customers do not switch in the second period, Eq. (7) is modified as follows : $q(v - p_1^A) - x_1^H + \delta\{q(v - p_2^{AH}) - x_1^H\}$

$$= q(v - p_1^B) - (1 - x_1^H) + \delta\{q(v - p_2^{BH}) - (1 - x_1^H)\},$$

$$(v - p_1^A) - x_1^L + \delta\{(v - p_2^{BR}) - (1 - x_1^L)\} = (v - p_1^B) - (1 - x_1^L) + \delta\{(v - p_2^{AR}) - x_1^L\}.$$

Thus, we have the following cutoffs:

$$x_1L = \frac{3(p_1^B - p_1^A) + \delta + 3}{2(\delta + 3)}, x_1H = \frac{(q(p_2^{BH} - p_2^{AH}) + 1)\delta + q(p_1^B - p_1^A) + 1}{2\delta + 2}.$$

By using the previously described profit functions for the second period, we can solve the optimization problem (≤ 8). However, in contrast to the case where s is small (s s_1), $p^{AH}{}_2$ and $p^{BR}{}_2$ are linearly dependent, as given in Eq. (17). Hence, the equilibrium is not uniquely determined by the FOC of Eq. (8). As we want to compare firms' equilibrium prices and profits for a small s with those for a large s, we consider the symmetric equilibrium for both a large s and a small s. In other words, we assume that $p^{AH}{}_2 = p^{BH}{}_2$. The results are obtained as follows.

Proposition 6 *Assume that the switching cost s satisfies* $s_1 \leq s \leq s_2$, *where*

$$s_2 = -\frac{((q^2 + 3)\alpha - 3)\delta^2 + 3\left[(q - 4)(q - 1)\alpha - 4\right]\delta - 9\left[1 + (q - 1)\alpha\right]}{2\delta\left[((q^2 - 3)\alpha + 3)\delta + 3((q^2 - 1)\alpha + 1)\right]}.$$

We also assume that $s \leq 1$. *In a symmetric equilibrium, the prices and the cutoffs are*

$\hat{p}A1 = \hat{p}B1 = 23(s2 - s)$,

$\hat{p}AL = p_2^{BL} = \frac{s+2}{3}$, $\hat{p}AH_2 = \hat{p}BH_2 = \hat{p}_2AR = \hat{p}_2BR = \frac{2s+1}{3}$,

$$\hat{x}_1^H = \hat{x}_1^L = \frac{1}{2}, \hat{x}_2^{AL} = \frac{s+2}{6}, \hat{x}_2^{BL} = \frac{4-s}{6}. \quad (21)$$

The total profit of firm $i \in \{A,B\}$ *in the equilibrium is* $\hat{\pi}^i = \hat{\pi}_1^i + \delta\hat{\pi}_2^i$, *where*

$$\hat{\pi}_1^i = \frac{\delta}{3}(\alpha q + (1 - \alpha))(s_2 - s),$$

$$\hat{\pi}_2^i = \frac{2(1 - \alpha)s^2 + (6q\alpha - 2\alpha + 2)s + 3q\alpha - 5\alpha + 5}{18}.$$

Note that $s \leq 1$ is the condition for the equilibrium $2AL \leq$ where $2BL$ $L \geq$-type customer switching occurs in the second period, i.e., $\hat{x}$ 1/2 and $\hat{x}$ 1/2 , which is directly obtained from Eq. (21). For $s > 1$, switching stops among both L- and H-type customers and BBPD is no longer applied.

s_2 is the switching cost at which the equilibrium prices in the first period are zero (i.e., ≤ 2 $\hat{p}_1^A = \hat{p}_1^B = 0$). Therefore, s s guarantees the nonnegativity of prices in the first period.

From the proposition, we observe that prices $\hat{p}^A_1$ and $\hat{p}^B_1$ are decreasing with s because $\partial \hat{p}_1^A/\partial s = \partial \hat{p}_1^B/\partial s = -2\delta/3 < 0$. Conversely, the prices in the second period increase with s, and we have the relationship $\hat{p}^{iR}_2 = \hat{p}_2^{iH} \leq \hat{p}_2^{iL}, i \in \{A, B\}$. The relationship between prices in the first and second periods is not simple, in contrast with the case of a small s.

Proposition 7 *We assume that iR* $s_1 \leq s \leq \min\{s_2,1\}$ *and consider a symmetric equiiR i librium. Then, we have* $\hat{p}_2 \leq \hat{p}_1^i$ *if* $s \leq \max\{s_1, \frac{2\delta s_2 - 1}{2(1+\delta)}\}$, *and* $\hat{p}_2 > \hat{p}_1$ *otherwise.*

It is easy to verify that $1 - s_1 = \frac{3(1-\alpha)(q-1)}{2q(\alpha(q-1)+1)} > 0$ holds for any α ($0<\alpha<1$) and any q ($q>1$). Thus, we can confirm that in our model, there is always a situation where switching occurs only for L-type customers. Note that it is possible that $s_2 > 1$. This case can be seen in the numerical examples that follow.≥ 2

For s s, the prices in the first period cannot decrease any further, and we have $\hat{p}_1^A = \hat{p}_1^B = 0$. Hence, the profit functions are modified as follows.

Corollary 2 *Assume that the switching cost s satisfies* $s \geq s_2$. *In a symmetric equilibrium, we have* $\hat{p}^A_1 = \hat{p}^B_1 = 0$ *and* $\hat{\pi}_1^A = \hat{\pi}_1^B = 0$. *The equilibrium prices and profits in the second period and the cutoffs are the same as those in the case where* $s_1 \leq s \leq s_2$. *The total profits of each firm in the equilibrium are* $\hat{\pi}^A = \hat{\pi}^B = \frac{2(1-\alpha)s^2 + (6q\alpha - 2\alpha + 2)s + 3q\alpha - 5\alpha + 5}{18}\delta.$

Table 1 Relationships between switching costs and equilibria for $s_1 \leq s_2$

Condition (C)	Condition (C)	
0 $s < s_1$	Switching occurs among both L- and H-type customers	Not available
s ≤ < $_1$ ≤ $s s$ s_2 s <1 2	Switching occurs among L-type customers only, and $\hat{p}_1^A = \hat{p}_1^B > 0$ Switching occurs among L-type customers only, and first-time free prices emerge, i.e., $\hat{p}_1^A = \hat{p}_1^B = 0$ No switching	
$s \geq_1$	occurs	

Table 2 Relationships between switching costs and equilibria for $s_1 > s_2$

≤

0≤ $s < s_1$	Switching occurs among both L- and H-type customers	Not available

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$s \leq$ <1 $_1$ s	Switching occurs among L -type customers only, and first-time free prices emerge, i.e., $\hat{p}_1^A = \hat{p}_1^B$
$s \geq 1$	= 0 No switching occurs

Condition (C) Condition $(\overline{C})$

We note that the first-time free prices (i.e., $\hat{p}_1^A = \hat{p}_1^B = 0$) are unique to the case where firms bear switching costs. When consumers bear switching costs, the prices in the first period are almost always positive (see the Appendix).[8]

Now, we examine the change in the firms' total profits in response to the change in switching costs. Interestingly, we find that when the switching cost is greater than or equal to s_2 , the firms' profits are increasing with s.

Proposition 8 *The total profits of each firm in equilibrium are nonincreasing with s if* $s_1 \leq s \leq s_2$, *whereas they are increasing with s if* $s \geq s_2$.

The decline of profits with s in the first period stops for $s > s_2$ because the prices in the first period remain at zero and therefore the increase of profits in the second period directly reflects the increase of the total profits. When $s > s_2$, the share of H-type customers remains the same regardless of the value of s, whereas the price in the second period increases with s, contributing to profits. Similarly, for L-type customers, poaching decreases as s increases, as shown in Eq. (21), resulting in an increase in the share of existing customers that are not poached. Moreover, the price offered to existing customers in the second period increases, and the decline in the poaching of customers relieves the burden of switching costs for firms. As a result, the profits in the second period increase with s. [8] More precisely, the prices in the first period are positive except for some cases in which Condition (C) is satisfied. Condition (C) , i.e., $q>3$ and $\alpha < (q-1q)(-23q+3)$, emerges very rarely. We can easily see that $(q-1q)(-23q+3)$ is maximized at $q=6$ and therefore the upper bound of α is $\frac{6-3}{(6-1)(2\cdot 6+3)} = 0.04$. In other words, Condition (C) only occurs for a relatively large q and an extremely small α.

We summarize the relationships between equilibrium prices and parameters q,
, and s. Depending on the combinations of parameters, the magnitude relationship 1 2 1 ≤ 21 ≥ 2 1 ≤ 2 between s and s , i.e., whether s s or s s , varies. If s s , and if Condition (C) holds, then switching of both types of customers occurs in equilibrium for 1 1 ≤ $s2 < s1$. However, once s reaches s , the switching of H-type customers stops. If s s , and if Condition (C) fails, i.e., if Condition (C) holds, then no equilibrium exists in which switching of both customers occurs. These relationships are summarized in Table 2 ≤ 1.

Note that we assume that 1 ≥ 2 s 1 for simplicity, although it can be more than one. If 1 s s , the price in the first period becomes zero after s exceeds s , which is larger than s_2 . Table 2 illustrates this situation.

3.3 Numerical examples

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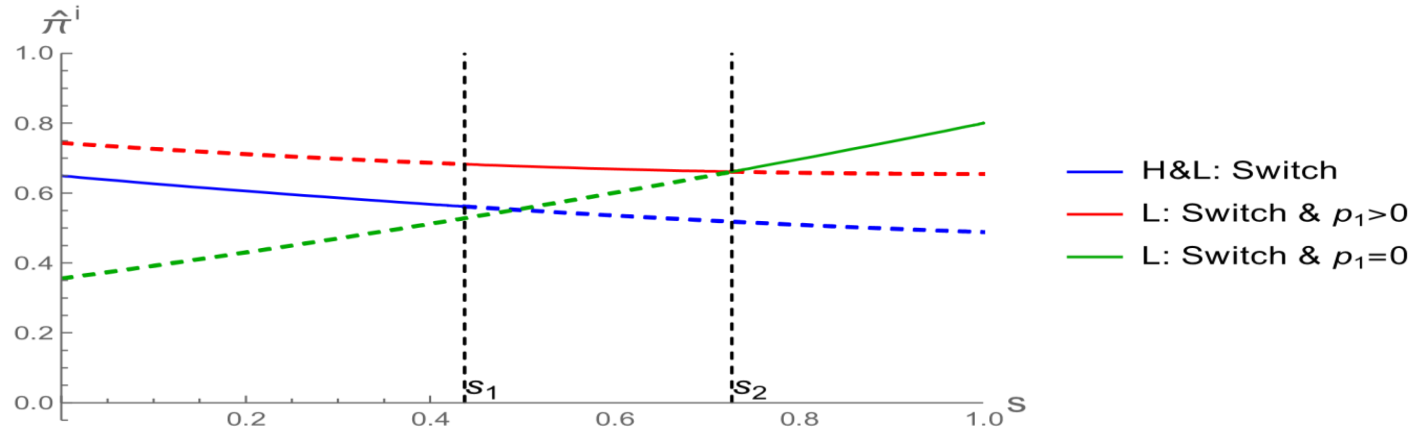
This subsection discusses the model from a business perspective by examining numerical examples. Unless otherwise noted, we assume that δ=1 . One of the characteristics of our model is that the poaching of *H*-type customers ceases as the switching costs increase, and then the initial free price of ˆ $p^{A}{}_{1}$ =0 appears. We conduct numerical experiments below to determine how firms' profits change during this process.

First, we set α= 0.2 and q=4 . This setting demonstrates the case where the pur-
chase volume of an *H*-type consumer is very high (i.e., four times larger than that of an *L*-type consumer), but the consumer population is dominated by *L*-type customers. In other words, the firms compete for a small number of prospective customers with high customer lifetime value. In this case, s_1 = 0.44 and s_2 = 0.73 (see Fig. 3). Although there is a jump at 1 ≤ 2 s = ^{s}≥1 , the total profits decrease with 2 s for s < s_1 and for s < s 1*A* s . However, when 1s s , the profit becomes increasing with 1 ≤ 2 s. This is because ˆp decreases for s < s and s < s s , offsetting the increase in profit in the second period. When s and the decrease in profit stops (see Figs. 3 and 4).



3 Switching costs and total profits (s_1 = 0.44, s_2 = 0.73, α = 0.2, q = 4)

$\geq s_2$, $\hat{p}_1^A = 0$

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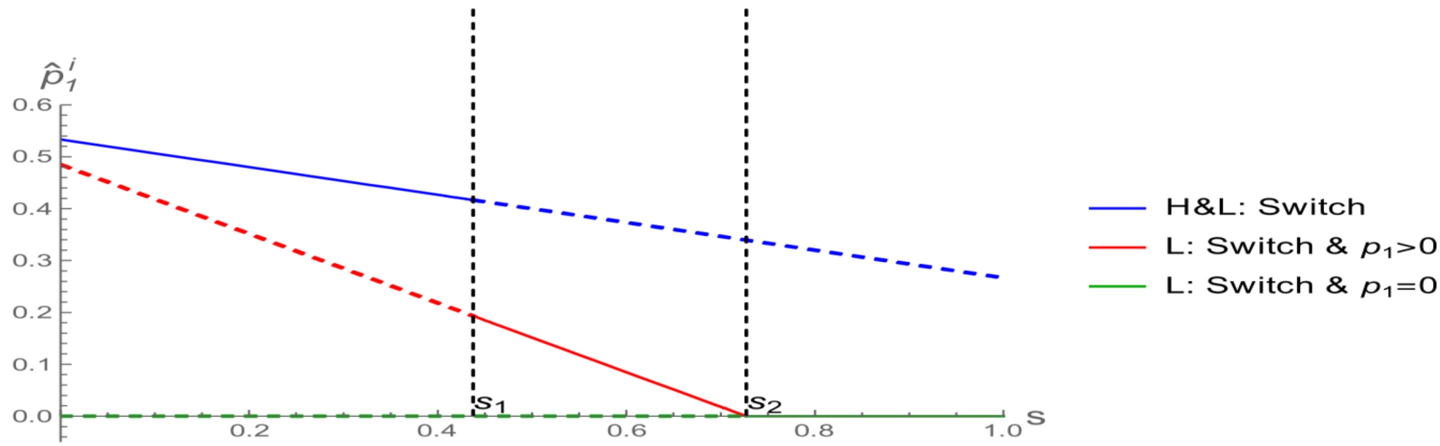

Fig. 4 Prices in the first period ($s_1 = 0.44, s_2 = 0.7, \alpha = 0.2, q = 4$)

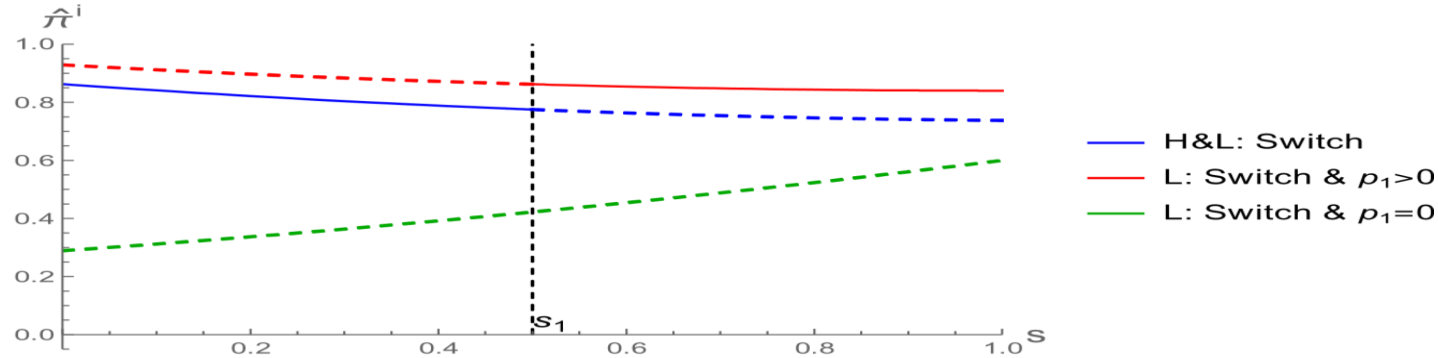

Fig. 5 Switching costs and total profits ($s_1 = 0.5, s_2 = 1.6, \alpha = 0.2, q = 2$)

Furthermore, the second-period prices p_2^{AH} and p_2^{BH} for *H*-type consumers become the limit prices at $s = s_1$, and the total profits exceed those in the case with $s = 0$ where no switching cost exists. Thus, when q is large, both firms attempt to protect their *H*-type customers even if α is relatively small. For a larger s, firms rationally decide to give up poaching *H*type customers, and to introduce the firsttime free price to secure *H*-type customers in the first period. It should be noted that firms are likely to increase their profits rapidly after making the first-time free offers. In this case, the decline of profits with s in the first period stops because the prices in the first period remain at zero; therefore, the increase in the profits in the second period directly reflects the increase in the total profits. Second, we set $\alpha = 0.2$ and $q=2$. Now, the gap between *L*-type and *H*-type purchase volumes is smaller than in the previous setting. In this case, we have $s_1 = 0.50$, $s_2 = 1.6 > 1$. Therefore, the first-time free price is not offered (see Fig. 5). In this case, the consumption volumes of consumers are relatively uniform throughout the market. Hence, firms do not have a strong incentive to retain *H*-type customers. Thus, they do not offer the first-time free price even if the switching costs are high. Similarly, they do not offer lower prices in the first period. Owing to this reluctance of firms to offer such price benefits to consumers, their total profits are relatively flat, regardless of the degree of the switching costs. However, because of lower competitive pressure, firms' profits remain higher than in the previous case.

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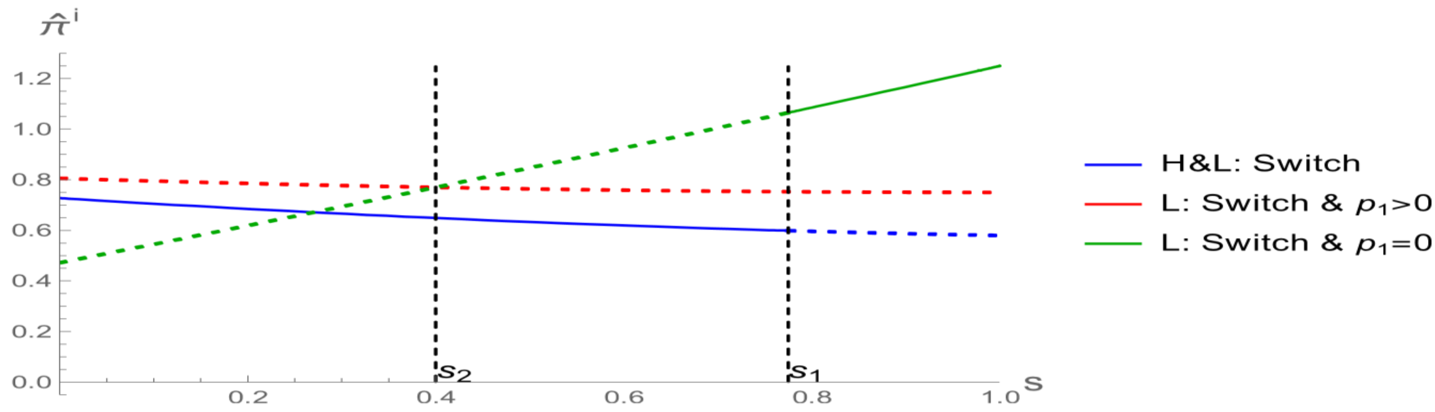
These two examples indicate that for switching costs to affect the behaviors of firms, there must be a difference in consumption between customer types. Analogous to the observation of Shin and Sudhir (2010) that BBPD matters when q is large, switching costs also matter when q is large. Third, we set α= 0.5 and q=4 . This is the case where the volume of purchases by *H*-type customers is high, and *L*-type and *H*-type customers equally occupy the market. In this case, we have s_1 = 0.775 and s_2 = 0.4 . Therefore, $s_1 > s_2$. This indicates that the first-time free pricing offer suddenly emerges with the occurrence of the limit price (see Fig. 6). In this setting, firms do not easily give up on attempting to poach the *H*-type customers of their rival because the purchase volume of these customers is large and their proportion of the market is high. Therefore, poaching continues even if s is large. However, once firms give up on the strategy of poaching *H*-type customers in the second period, they will focus on acquiring more *H*-type customers in the first period. In particular, as mentioned above, this market is very attractive to firms because of the large α and q. Therefore, firms aggressively offer first-time free



6 Switching costs and total profits ($s_1 = 0.775$, $s_2 = 0.4$, $\alpha = 0.5$, $q = 4$)

prices when they cease poaching. As the burden of poaching under a high s and large α is high, the firms' profits are significantly improved after they cease poaching *H*- type customers.

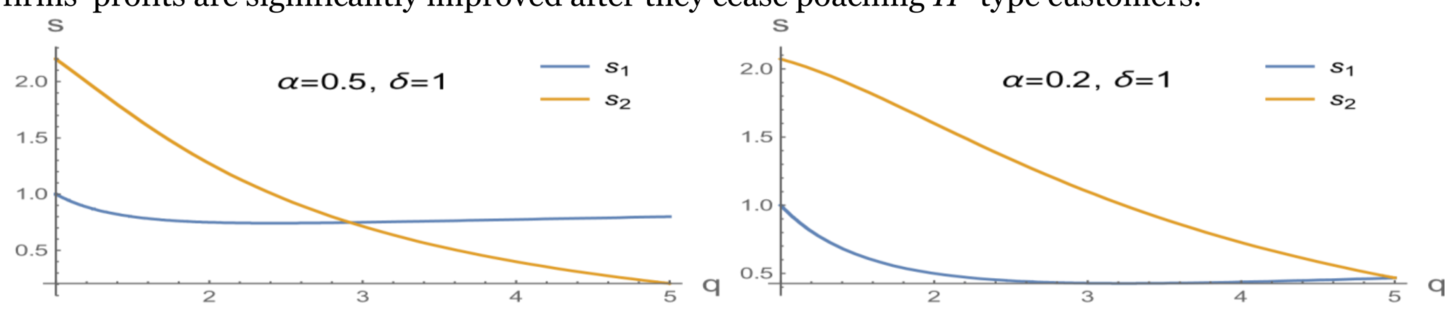


Fig. 7 Purchase quantities q and switching costs s_1 and s_2

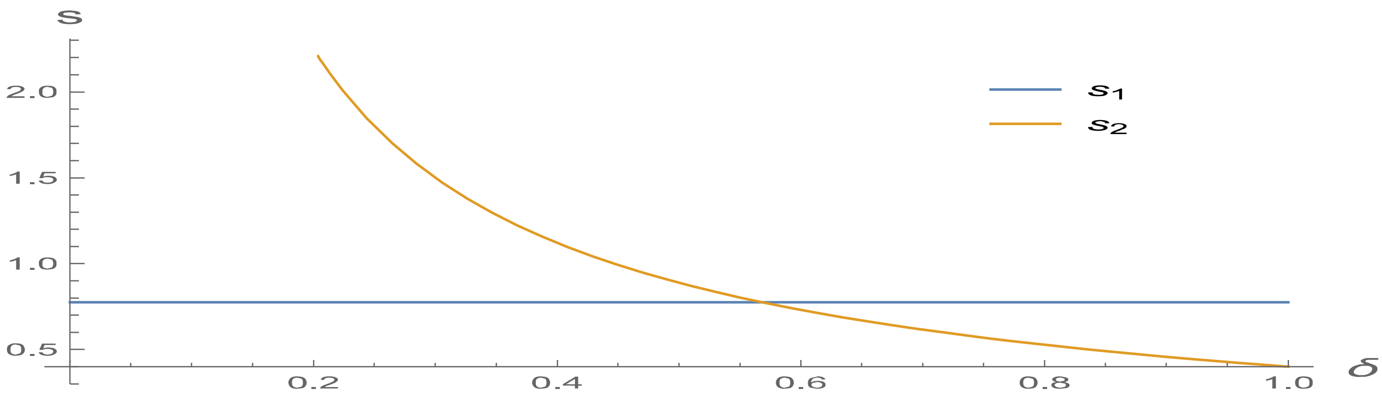


Fig. 8 Discount factors δ and switching costs s_1 and s_2 (q=4 , α= 0.5)

In the case of Fig. 6, the first-time free offer appears suddenly. We want to observe at what purchase volume q such a phenomenon occurs. Figure 7 (left) shows the boundary points s_1 and s_2 when α is fixed at α= 0.5 and the purchase quantity q is varied as a variable. The figure shows that s_1 exceeds s_2 around q= 2.92 . Conversely, when α= 0.2 , s_2 does not fall below s_1 even for q=4 (see Fig. 7 (right)). Thus, a larger α reduces s_2 drastically.

As noted previously, we set δ=1 in this subsection. Now, we would like to observe how δ affects the boundary point s_2 . When α= 0.5 and q=4 , we have $s_2 = -13(\delta^2 - 24\delta - 45)(38\delta^2 + 102\delta)$. Figure 8 shows how s_2 varies in terms of . We note that s_1 does not depend on , and that s_2 does not exist for δ=0 . The figure shows that s_2 decreases rapidly with . That is, the more value that the firms attach to the second period, the lower is the boundary point s_2 . In other words, the price setting of firms varies depending on whether they adopt a short-term or a longterm perspective.

3.4 Social impact of BBPD

Now, we investigate how BBPD affects CS and SW. We also determine how CS and SW change for large and small switching costs, corresponding to the above numeri1 2 3 ≤ 1cal example 1 ≤ ≤ 2 (δ =21)≤ . First, we consider CS. Let *CS* ,*CS* , and *CS* be CS for *s s* , *s s s* , and *s s* , respectively. We can state the following proposition, noting that CSs are illustrated in Fig. 9 for the parameters α= 0.2 and q=4 , as per the setting in the previous subsection.

Proposition ≤ ≤ 1 **9** *Suppose* 1 ≤ ≤ 2*that* 0≤ $s_1 \le s_2$ <1 . *CS increases for each range of* 0 *s s and s s s to the extent that firms decrease their profits, whereas CS decreases for s* ≥ s_2 *. Specifically,* $\frac{\partial CS_1}{\partial s}>0$, $\frac{\partial CS_2}{\partial s}>0$, *and* $\frac{\partial CS_3}{\partial s}<0$ *for* 0≤ *s* ≤1.

2 ≤Next, consider SW. Let W_1 , W_2 , and W_3 be SW for $s \le s_1$, $s_1 \le s \le s_2$, and *s s* , respectively, defined by the sum of firms' profits and the CS. That is, $W_\ell = \hat{\pi}^A + \hat{\pi}^B + CS_\ell$ for ℓ= 1,2,3 , according to the size of *s*. Note that W_2 and W_3 are the same. This is because the increase in firms' profits is offset by the decrease in 1 ≤ ≤ 2 2 ≤ CS, and the cutoffs are the same for the cases of *s s s* and *s s* . We can state the following proposition.

Proposition 10 *When switching costs are sufficiently large, SW is an increasing function of s. Moreover, SW achieves its maximum value at* $s = 1$ *, which is the upper bound at which BBPD occurs.*

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SW rises with switching costs when they are sufficiently high because the transportation and switching costs incurred by the firms from poaching customers—both of which involve SW losses—decline as poaching decreases. Thus, we find that a larger s is beneficial from the perspective of SW, although it is detrimental to CS. For a small s, poaching and switching continues to occur frequently in the second period. Therefore, an increase in the unit switching cost can increase the total switching costs in society,

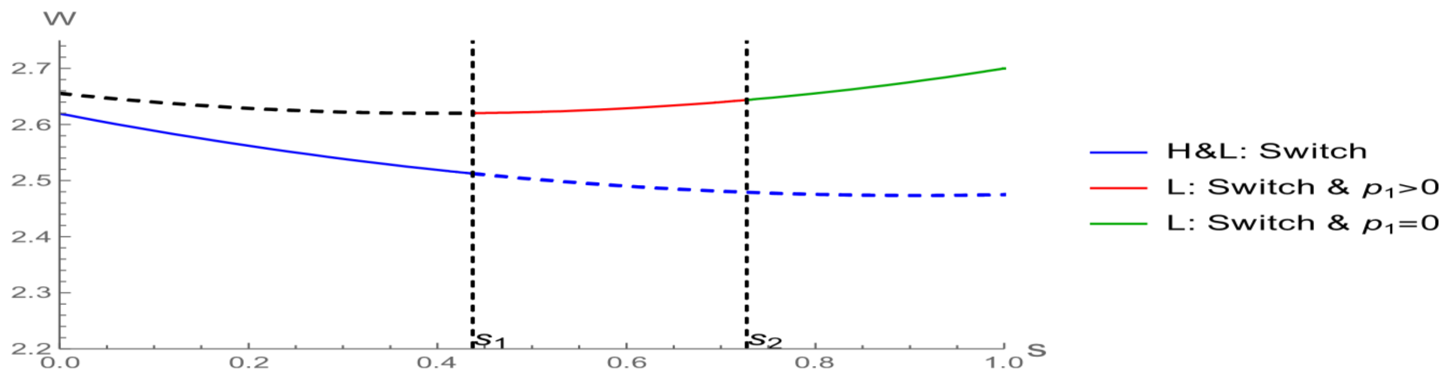
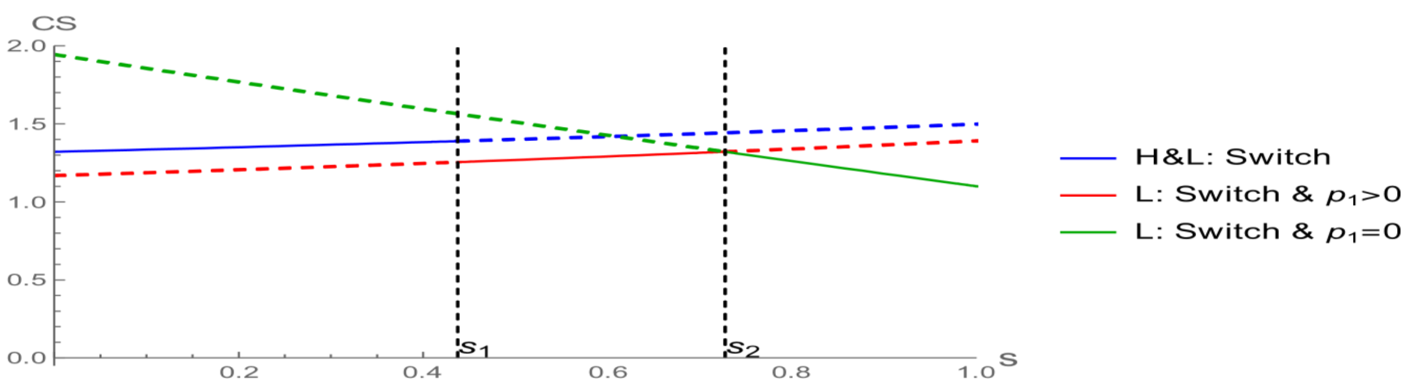


Fig. 10 Switching costs and SW ($\delta = 1$, $\alpha = 0.2$, $q = 4$)

resulting in a decline of SW. As shown in Fig. 10, representing SW for the parameters α= 0.2 and q=4 , switching cost s is a negative factor for SW in $s < s_1$, but it becomes a positive factor in $s > s_1$.

4 Comparison with benchmarks



9 Switching costs and CS ($\delta = 1$, $\alpha = 0.2$, $q = 4$)

In this section, we compare the results of our model with those of benchmark models. We have confirmed that no equilibrium in pure strategies exists under uniform pricing (UP). Instead of UP, we consider a situation where a firm sets prices based on purchase history but does not engage in pricing discrimination between L- and H-type customers based on quantity demanded. We refer to this situation as partial uniform pricing (partial UP).

4.1 Partial uniform pricing

The timing of the game with partial UP in each period is as follows. In the first period, firms A and B simultaneously set prices p^A_{1PU} and p^B_{1PU} . In the second period, each firm $i \in \{A,B\}$ sets prices p^{iO}_{2PU} for its own customers, regardless of the type of customer, and a poaching price p^{iR}_{2PU} for its rival's customers. Consumers decide from which firm to purchase the goods. Having knowledge of this process, the firms determine prices to maximize their profits throughout the entire period. Let $x_2^{ik}{}_{PU}$ ($i \in \{A,B\}$, $k \in \{L,H\}$) denote the cutoff location of k-type consumers who bought from firm i in the first period and who are indifferent between buying good A or good B in the second period. The locations $x_2^{Ak}{}_{PU}$ and $x_2^{Bk}{}_{PU}$ can be given for k as follows:

$pBR - pAO\ Qk + 1 \quad pBO - pAR\ Qk + 1\ Ak \quad (2PU\ 2PU) \quad Bk \quad (2PU\ 2PU)\ x2PU = 2$

and $x2PU = 2$.

The following problems are solved on A's turf in the second period by firms A and B, respectively:

$\max AO\ \pi_2 AA_{PU} = pAO_{2PU}\alpha q x_2 AH_{PU} + pAO_{2PU}(1-\alpha)x_2 AL_{PU},\ p2PU$

$\max BR\ \pi 2BRPU = pBR2PU(\alpha q(x1HPU - x2AHPU) + (1-\alpha)(x1LPU - x2ALPU))$

$p2PU$

$-s[\alpha(x1HPU - x2AHPU) + (1-\alpha)(x1LPU - x2ALPU)],$

where $x_1^k{}_{PU}$ is the cutoff of k-type consumers who are indifferent between buying from A or B in the first period. By solving the FOCs, we obtain the following optimal prices:

$AO \quad 1 + s + 2x_1^L{}_{PU} + (-1 - s + q(1 + s + 2x_1^H{}_{PU}) - 2x_1^L{}_{PU})$

$p2PU = \quad 3((q2-1)+1)$ and

$BR \quad -1 + 2s + 4x_1^L{}_{PU} + (1 - 2s + q(-1 + 2s + 4x_1^H{}_{PU}) - 4x_1^L{}_{PU})$

$p^{2PU} = 3((q^2-1)+1)$.

On B's turf, the profit-maximization problems of firms A and B on B's turf in the second period are, respectively:

$\max AR\ \pi 2ARPU = pAR2PU(\alpha q(x2BHPU - x1HPU) + (1-\alpha)(x2BLPU - x1LPU))$

$p2PU$

$-s[\alpha(x2BHPU - x1HPU) + (1-\alpha)(x2BLPU - x1LPU)],$

$\max BO\ \pi 2BBPU = pBO2PU\alpha q(1 - x2BHPU) + p2BOPU(1-\alpha)(1 - x2BLPU).$

$p2PU$

By solving the FOCs, we obtain the following optimal prices on B's turf:

$3 + 2s - 4x1^L PU + (-3 - 2s + q(3 + 2s - 4x1^H PU) + 4x1^L PU)$

AR

$p2PU = \quad 3((q2-1)+1)$ and

$BO \quad 3 + s - 2x1LPU + \alpha(-3 - s + q(3 + s - 2x^H_{PU}) + 2x^L_{1PU})1$

$p2PU = \quad 3((q2-1)+1)$.

Thus, ≡we obtain the profit in the second period of firm i ($i \in \{A,B\}$) :

$\pi 2^i PU \quad \pi^{ii}_{2PU} + \pi^{iR}_{2PU}$.

The location $x_1{}^k{}_{PU}$ can be determined for $k\in\{L,H\}$ as follows:

$Qk(v-pA1PU)-x1kPU+\delta(Qk(v-pBR2PU)-(1-x1kPU))$

$= \quad Qk(v-pB1PU)-(1-x1kPU)+\delta(Qk(v-pAR2PU)-x1kPU)$.

$_{1PU} = \frac{}{2} + \frac{(p^A - p^B{}_{PU}) - ?(p_2^{AR} - p^{BR}_{2PU})}{2(-1+?)}$ Thus, $x^{k\,1\,1PU\,1\,PU} Q^k$. By substituting p^{AR} and p^{BR}, we have $2PU \quad 2PU$

$\frac{3+?-3Q^k(p^A_{1PU}-p^B_{1PU})}{}$.

$x1kPU = \quad 6+2\delta$

Each firm determines the optimal price in the first period as follows:

$\max A\ \pi PUA = \pi 1APU + \delta(\pi 2AAPU + \pi 2ARPU), \max_B \pi PUB = \pi 1BPU + \delta(\pi 2BBPU + \pi 2BRPU)$,

$p1PU \quad p1PU$

where $^B_{1PU} = p^B_1$ (A A ( ) . We have the prices, the cutoffs, and) and

$\pi_{1PU} = p_{1PU} \quad \alpha q x^H_{1PU} + (1-\alpha)x^L_1 PU \ \pi PU \quad \alpha q(1-x_1{}^H{}_{PU})+(1-\alpha)(1-x_1{}^L{}_{PU})$

the total profits of the firms in equilibrium as follows:

$A \quad \hat{}B \quad = ((q-1)+1)(\delta - 2s\delta + 3)$, $x\hat{}1LPU = x\hat{}1HPU = 1/2$, $p\hat{}1PU = p1PU \quad 3((q2 - 1)\alpha + 1)$

$AO \quad \hat{}BO = (1 + (q-1))(2+s)$, $p\hat{}AR2PU = p\hat{}BR2PU = \frac{(q-1)(2s+1)\alpha + 2s + 1}{3((q^2-1)\alpha + 1)}$,

$p\hat{}2PU = p2PU \quad 3((q2-1)+1)$

$AL \quad 2+s+(q-1)(2+3q+s)\alpha\ \hat{}AH = 3+q(-1+s)+(q-1)(3+q(2+s))\alpha$,

$x\hat{}2PU = \quad 6((q2-1)+1) \quad , x2PU\,6((q2-1)\alpha+1)$

$BL\ 4-s+(q-1)(4+3q-s)\alpha\ \hat{}BH = 3+q(1-s)-(q-1)(-3+q(-4+s))\alpha$, $x\hat{}2PU = 6((q2-1)\alpha+1)$,

$x2PU\ 6((q2-1)\alpha+1)$

$\pi\hat{}A = \pi\hat{}PUB = \frac{(1+\alpha(q-1))^2(9+2(4-2s+s^2)\delta)}{18((q^2-1)\alpha+1)}$.

PU

It is easily confirmed that if $s \leq 1$, then the existence of an equilibrium where switching occurs among both L- and H- type customers in the second period is guaranteed. We obtain CS and SW under partial UP as follows:

$36(\delta + 1)v(\alpha(q-1)+1)\left(\alpha\left(q^2-1\right)+1\right) + 2\alpha^2(q-1)^2(\delta(s(s+4)-17)-18)$
$-\alpha(q-1)(\delta(9q-4s(s+4)+77)+9(q+9)) + \delta(2s(s+4)-43) - 45$

$CSPU =,$

$36((\ 2-1)+1)$

q

$-9-9\alpha\left(q^2-1\right)+36(\delta+1)v(\alpha(q-1)+1)\left(\alpha\left(q^2-1\right)+1\right)$
$\left(\alpha(q-1)(2\alpha(q-1)+9q+13)+8s(\alpha(q-1)+1)^2-10(\alpha(q-1)s+s)^2+11\right)$

$-\delta$

SWPU =.

$((q2-1)+1)$

4.2 Impacts of BBPD on profits and welfare

Now, we compare the profits and welfare under BBPD with those under partial UP.

Proposition 11 *The following relationship holds for the total profits of each firm under BBPD and partial UP.*

(a) *Suppose that Condition* (C) *and* $s < s_1$ *hold, i.e., switching occurs among both L- and H-type customers in the second period. The total profit of each firm is larger under BBPD than under partial UP if and only if* ≥ 1 $s < 1/2$.

(b) *Suppose that* $s\ s$ *holds, i.e., switching occurs only among L-type customers in the second period. The total profit of each firm is larger under BBPD than under partial UP.*

Part (a) of Proposition 11 shows that when switching costs *s* are small, differentiating prices between high- and low-demand customers succeeds in eliciting more demand and positively affects firm profits. However, the burden on poaching firms becomes heavier as *s* increases and exceeds 1/2, and thus it is no longer worthwhile to aggressively price discriminate by differentiating prices between *L*- and *H*-type customers. In fact, we can confirm that $\hat{p}^{iH}_2 < \hat{p}^{iO}_{2PU} < \hat{p}^{iL}_2$ and $\hat{p}^{jR}_2 = \hat{p}^{jR}_{2PU}$ $(i, j \in \{A, B\})$, indicating that price competition in the *H* segment, which has a greater impact on profits in the second period, is more intense under BBPD. However, when there is no ≥ 1 poaching of *H*-type customers under BBPD (i.e., *s s*), firms bear lower switching costs under BBPD, whereas firms continue to poach both types of customers under partial UP. Therefore, the profits under partial UP are smaller than those under BBPD, which is demonstrated in part (b) of Proposition 11. From the above, when firms poach both types of customers, and the switching cost is relatively large, firms have an incentive to engage in partial UP rather than BBPD, but otherwise, firms are better off conducting BBPD.

Proposition 12 *CS is lower under BBPD than under partial UP.*

Generally, the effect on CS is the opposite to the effect on firm profits. In other words, when profits are higher under BBPD than under partial UP, CS is likely to be smaller under BBPD than under partial UP, and vice versa. The exception is when poaching of both *L*- and *H*-type customers occurs, and switching costs are high (i.e., $s > 1/2$). In this case, both profits and CS are higher under partial UP than under BBPD. The following occurs in this case. Price competition is generally more intense under BBPD, and switching costs are higher. Hence, profits are lower under BBPD. However, the number of poached *H*-type consumers who benefit from lower prices under BBPD is not large due to the high switching costs, and thus CS is not larger under BBPD than under partial UP.

Proposition 13 *The following relationship holds for SW under BBPD and partial UP.*

(a) *Suppose that Condition* (C) *and* $s < s_1$ *hold. SW is lower under BBPD than under partial UP.* ≥ 1

(b) *Suppose that* $s\ s$ *holds. SW is lower at BBPD than at partial UP if and only if* $\alpha < (q-2)/2(q-1)$.

Note that from Proposition 12, CS is always higher under partial UP. Proposition 13 indicates that the impact of BBPD on CS is generally larger than that on profits. However, when firms stop poaching *H*-

type customers and the proportion α of them is large, firms earn larger profits because competition is lessened under BBPD, and the impact is stronger. As a result, SW is also higher under BBPD in such cases (see (b)).

5 Conclusion

This paper considers a two-period BBPD model in which two types of consumers with different demands exist, and switching costs are incurred when the consumers switch firms. We assume that firms compensate the customers for the switching costs incurred. As switching costs increase from zero, competition in the first period becomes fiercer than that of the second period; therefore, prices in the first (second) period decline (rise). This shifting of competition to the first period occurs because poaching customers from the rival firm and compensating them for their switching costs becomes burdensome as switching costs increase. Therefore, firms try to gain more market share in the first period and to secure profits in the second. When switching costs become larger and exceed a certain threshold, firms give up attempting to poach their competitor's H-type customers. The prices for L-type customers remain above the prices for H-type customers; hence, the rival firm keeps poaching L-type customers for a while. In other words, firms preferentially protect their H- type customers.

When switching costs become larger, "first-time free" offers emerge in equilibria. To our knowledge, this result is not presented in the literature on BBPD in a differentiated market; however, it is common practice in reality among firms seeking profits in the future ahead of current sales. Note that "first-time free" offers do not occur in a model with only one type of consumer. Even if two types of consumers exist, the offers do not occur as long as switching occurs for both L- and H-type customers. Note also that the first-time free price is specific to schemes in which firms bear switching costs. In the case where consumers bear switching costs, the prices in the first period are almost always positive. Surprisingly, the emergence of "first-time free" offers allows firms to improve their profits. As the prices and firms' profits in the first period decrease when the switching costs increase, the zero prices, i.e., the first-time free offers, establish the moment at which the escalation of competition stops. Simultaneously, SW rises with switching costs when such costs are sufficiently high because the transportation and switching costs incurred due to poaching customers, both of which involve a loss of SW, decline as poaching decreases. We conclude this paper by discussing possible extensions of the model in future research. First, we do not consider asymmetric equilibria in this paper. Given that multiple equilibria can exist with higher switching costs, it would be intriguing to investigate whether "first-time free" offers function similarly in asymmetric equilibria to how they function in the symmetric equilibrium. Second, our model assumes that firms are horizontally differentiated but symmetric in other aspects, such as product quality. Extending our model to include competition between asymmetric firms would be interesting. Third, our research focuses on BBPD with switching costs, but we have not compared it with uniform pricing. We have conducted some analyses of uniform pricing in our setting and found that no equilibrium in pure strategies exists under uniform prices. Therefore, another potential area for future research is to find equilibria with mixed strategies under uniform pricing and compare them with BBPD. Finally, we would like to consider whether firms might contemplate adopting other

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strategies to encourage *H*-type consumers to switch in the second period. One option is to introduce second-degree price discrimination when consumer information is unavailable. The idea behind this strategy is to separate the poaching and first-period prices by *L*- and *H*-type customers and to let consumers self-report their demand. However, when separate prices are offered, it is easy to see that *L*-type consumers are motivated to falsely report that they are *H* types. This is because the prices are always smaller than those for *L*-type consumers in our setting.[9] Therefore, a mechanism to control consumer motivation (e.g., pay-in-advance or nonlinear pricing) is needed to introduce second-degree price discrimination. This is an interesting research question, but it is beyond the scope of this paper and will be the subject of future research.

Appendix

Proof of Proposition 1

We obtain the prices, cutoffs, and profits in equilibrium by solving the FOCs of the firms' profit-maximization problems in the first period.

Now, we explore the conditions of the parameters that guarantee the existence of equilibria. As the cutoffs $\hat{x}_2^{AL}$ and $\hat{x}_2^{AH}$ must exist within *A*'s turf, $\hat{x}_2^{AL} \leq \hat{x}_1^L$ and $\hat{x}^{AH} \leq \hat{x}_1^H$ must hold. Although similar conditions must hold for *B*'s turf, it is suf-$_2$
ficient to consider them for *A*'s turf. ≤

First, consider $\hat{x}_2^{AH} \leq \hat{x}_1^H$. In fact, this condition is equivalent to $\hat{p}^{BR}_2 \; \hat{p}^{AH}_2$, the condition where the rival's poaching price is lower than the price of the current firm, ≤ and poaching by firm *B* occurs on *A*'s turf. Note that when $\hat{p}^{BR}_2 \; \hat{p}^{AH}_2$, we have

[9] Consider separating prices in the first and second periods. Letting the first- and second-period poaching prices be p^{AL}_1, p^{AH}_1, p^{ARL}_2, and p^{ARH}_2 (the same for firm *B*), we obtain

$AL \quad BL \quad 3+\delta-2\delta s \quad AH \quad BH \quad 3+\delta-2\delta qs$

$p_1 = p_1 =, p_1 = p_1 = \quad \frac{}{3} \quad \frac{}{3\sim}$

$pARL_2 = pBRL_2 = \quad , \quad \frac{1+2\cdot}{} \quad \frac{1+2\cdot}{}. \quad pARH_2 = pBRH_2 =$

$3 \quad 3q$

In both cases, the price for H–type customers is lower than that for the L–type customers.

$\hat{x}_2^{AL} \leq \hat{x}_1^L$ because it follows from $\hat{p}^{AL}_2 - \hat{p}^{AH}_2 = {}^{q_2 - q_1} > 0$ that $\hat{p}^{BR}_2 \leq \hat{p}^{AL}_2$. Now, we evaluate $\hat{p}^{BR}_2 - \hat{p}^{AH}_2$ and find the ranges of parameters that satisfy $\hat{p}^{BR}_2 - \hat{p}_2^{AH} \leq 0$:

$$\hat{p}^{BR2} - \hat{p}_2^{AH} = \frac{2q((q-1)\alpha+1)s-(q-1)(2q+3)\alpha+(q-3)}{6q((q^2-1)\alpha+1)}. \qquad (22)$$

As the denominator of Eq. (22) is positive for $q>1$, the sign of the equation depends on the numerator. Note that the coefficient of s in the numerator is positive and that we write s_1 as s when $\hat{p}^{BR}_2 = \hat{p}^{AH}_2$ holds, as provided in Eq. (≤ 10).

First, consider the case where $(1<)q \; 3$. The constant term for s in the numerator of Eq. (22) is clearly negative. Therefore, $\hat{p}^{BR}_2 - \hat{p}_2^{AH} \leq 0$ iff $0 \leq s \leq\geq s_1$. Next, consider the case where ≤ ≤$q1>3$. When $(q-1)(2q+3)\alpha$ $q-3$ holds, $\hat{p}^{BR}_2 - \hat{p}_2^{AH} \leq 0$ if and only if 0 s s. When $(q-1)(2q+3)\alpha<q-3$ holds, the numerator of Eq. ($BR2$

22) is positive, which means that $\hat{p} \leq_2^{BR} - \hat{p}_2^{AH} > 0 \leq 1$ for any s. In summary, $\hat{p} - \hat{p}_2^{AH} \leq 0$ holds if Condition (C⇒ and 0 s s hold. In particular, if s is fixed
at 0, Condition (C) guarantees $\hat{p}^{BR}{}_2 - \hat{p}_2^{AH} \leq 0$.

Now, we must confirm that the prices in equilibrium are nonnegative. It is trivial from Eqs. (12)–(14) that the prices in the second period are positive. However, it is not obvious whether $\hat{p}_1^A$ and $\hat{p}_1^B$ are nonnegative.

From Eq. (i1 ≤ 11), $\hat{p}_1^i$ $(i \in \{A, B\})$ is decreasing with ≥ s. Assume for contradiction ≥ that $\hat{p}$ 0≤ . This assumption is equivalent to s 1⁄2+(3⁄2δ) and therefore s 2 because δ 1 . When s =2 , we obtain

$$\hat{p}^{BR}{}_2 - \hat{p}^{AH}{}_2 = \frac{2q^2\alpha + (5q-3)(1-\alpha)}{6q((q^2-1)\alpha+1)} > 0.$$

Moreover, ≥ $\hat{p}^{BR}{}_2 - \hat{p}^{AH}{}_2$ is increasing with $BR2s$. Thus, we have $\hat{p}_2^{BR} - \hat{p}^{AH}{}_2 > 0$ for i1 any s 2. This contradicts the inequality $\hat{p} - \hat{p}_2^{AH} \leq 0$ proved above. Thus, $\hat{p} > 0$ holds. QED.

Proof of Proposition 4

From Eqs. (11) and (14) , we have

$$\hat{p}^{iR}{}_2 - \hat{p}_1^i = \frac{(\alpha(q-1)+1)(2s(\delta+1)-2-\delta)}{3\ \alpha(q^2-1)+1}(\quad) \quad .$$

Because 0<α<1 and $q > 1$, $\hat{p}_2^{iR} - \hat{p}_i^i \leq 0$ for $s \leq \frac{2+\delta}{2(1+\delta)}$, and $\hat{p}^{iR}{}_2 - \hat{p}^i{}_1 > 0$ for

s > 2(21++$\delta\delta$) . QED.

Proof of Proposition 5

The partial derivative of the total profit of firm ≤ ≤ 1 $i \in \{A,B\}$ with respect to s such that
0 s s is

$\partial\hat{\pi}^i$ 8 + 16(q− 1)+ 8α^2(q− 1)² −8 − 7α(q− 1)+ α^2(q− 1)² − 9αq(q− 1)
= — ∂s+ δ. ∂s 36((q² − 1) + 1) 36((q² − 1)α + 1)

The coefficient of s is obviously positive, which means that $\partial\hat{\pi}^i / \partial s$ increases with s. It is easily confirmed that when $s = s_1$, $\partial\hat{\pi}^i / \partial s = \frac{-(1-\alpha)(q-1)(4+\alpha(-4+q+3q^2))}{12q((q^2-1)\alpha+1)}\delta < 0$. Therefore, $\partial\hat{\pi}^i / \partial s$ ≤ ≤ i decreases with s. QED.

<0 for 0 s s_1. Thus, $\hat{\pi}$

Proof of Proposition 7

From Proposition 6, we have

$\hat{p}^{iR}{}_2 - \hat{p}^i{}_1$ = 2s3+ 1 + 23δs− 23δs2 = 13(2(1 +δ)s+ 1 − 2δs2),

and therefore

$$p^{iR}{}_2 > p^i{}_1 \Leftrightarrow s > \frac{2\delta s_2 - 1}{2(1+\delta)}.$$

Because $\frac{2\delta s_2 - 1}{2(1+\delta)} < \frac{2\delta s_2 + 2s_2 - 1 + 1}{2(1+\delta)} = s_2$, we have $p^{iR}{}_2 > \hat{p}_1^i$ if $\max\{s_1, \frac{2\delta s_2 - 1}{2(1+\delta)}\} < s \leq s_2$,
combining $s_1 \leq s \leq s_2$. The relationship between $\frac{2\delta s_2 - 1}{2(1+\delta)}$ and s_1 depends on the parameters.10 QED.

Proof of Proposition 8

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The partial derivatives of the total profits with respect to s such that $s_1 \le s \le s_2$ are

$$\frac{\partial \hat{\pi}^A}{\partial s} = \frac{\partial \hat{\pi}^B}{\partial s} = \frac{2(s-1)(1-\alpha)\delta}{9}.$$

This indicates that the value of the derivatives is nonpositive for $s \le 1$. Because the condition for the existence of the equilibrium, i.e., $\hat{x}_2^{AL} \le 1/2 \le \hat{x}_2^{BL}$, is $s \le 1$, the total profits are nonincreasing functions of s.

For $s \ge s_2$, we have the following partial derivatives:

$$\frac{\partial \hat{\pi}^A}{\partial s} = \frac{\partial \hat{\pi}^B}{\partial s} = \frac{2(1-\alpha)s+(3q-1)\alpha+1}{9}\delta.$$

Because $0 < \alpha < 1$, $0 < \delta \le 1$, and $q > 1$, the value of the derivatives is positive. Thus, the total profits of the firms increase with the switching costs. QED.

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10 For example, the values of s_1, s_2, and $\frac{2\delta s_2 - 1}{2(1+\delta)}$ are 0.47, 0.60, and −0.01, respectively, for $(\alpha, q, \delta) = (0.5, 5, 0.8)$. They become 0.50, 2.78, and 0.60 for $(\alpha, q, \delta) = (0.2, 2, 0.5)$, and s_1 becomes less than $\frac{2\delta s_2 - 1}{2(1+\delta)}$

Proof of Proposition 9

We provide the definition of CS and the results of the calculation.

$$CS_1 = \alpha\left[\int_0^{x_1^H} \{q(v - p_1^A) - x\}dx + \int_{x_1^H}^1 \{q(v - p_1^B) - (1 - x)\}dx\right]$$

$$+(1-\alpha)\left[\int_0^{x_1^L} \{(v - p_1^A) - x\}dx + \int_{x_1^L}^1 \{(v - p_1^B) - (1 - x)\}dx\right] + \alpha\delta\left[\int_0^{x_2^{AH}} \{q(v - p_2^{AH}) - x\}dx + \int_{x_2^{AH}}^{x^H} \{q(v - p_2^{BR}) - (1 - x)\}dx\right.$$

$$\left.+\int_{x^H}^{x_2^{BH}} \{q(v - p_2^{AR}) - x\}dx + \int_{x_2^{BH}}^1 \{q(v - p_2^{BH}) - (1 - x)\}dx\right]$$

BH

1 2

$\int_0^{x_2} AL$

$+(1-\alpha)\delta[\{(v-p^{AL_2})-x\}dx$

$+\int_{x_2^{AL}}^{x_1^L}\{(v-p_2^{BR})-(1-x)\}dx+\int_{x_1^L} xBL$

$+ \quad ^1 \{(v-p^{BL})-(1-x)\}dx]$

2

$\{(v-p^{AR_2})-x\}dx$

2

$\int xBL$

$_2$

$-90-\alpha(-1+q)(18(9+q)+\delta(127+45q-8s(4+s)))+72v$

$$=\frac{\begin{array}{c}+72\alpha(1+\delta)(-2+q+q^2)v+2\delta(-43+2s(4+s)+36v)\\+\alpha^2(-1+q)^2(72(-1+v+qv)+\delta(-41+4s(4+s)+72(1+q)v))\end{array}}{72((q^2-1)\alpha+1)}.$$

$x1H$

$CS_2 = \int [\{q(v-p^{A_1})-x\}dx$

0

1 x_1L

$+H \int \{q(v-p^{B_1})-(1-x)\}dx]+(1-\alpha)[\int \{(v-p^{A_1})-x\}dx$

x_1 0

$\int_{x_1^L}^1\{(v-p_1^B)-(1-x)\}dx]+\alpha\delta[\int_0 x1H$

$+\{q(v-p^{AH_2})-x\}dx$

$+\int_{x_1^H}^1\{q(v-p_2^{BH})-(1-x)\}dx]$

$+(1-\alpha)\delta[\int_0^{x_2^{AL}}\{(v-p_2^{AL})-x\}dx+\int_{x_2^{AL}}^{x_1^L}\{(v-p^{BR_2})-(1-x)\}dx$

$\int_{x_1^L} x2BL$

$+\{(v-p^{AR_2})-x\}dx$

$+\int_{x_2^{BL}}^1\{(v-p_2^{BL})-(1-x)\}dx]$

$3(1+\delta)(9(-5+4v)+\delta(-43+2s(4+s)+36v))+2\alpha^2(54(-1+q)^2(-1+v+qv)$

$+3\delta(-1+q)(35-s(4+s)-q(25+s(4+s))-36v+24q^2v)+\delta^2(-51+3s(4+s)$

$+q(36-54v)+54v+18q^3v-q^2(-5+s(4+s)+18v)))+\alpha(27(-1+q)(-9-q+4(2+q)v)$

$+6(79-2s(4+s)-72v+12q(-5+3v)+q^2(-11+s(4+s)+24v))$

$$= \frac{+\delta^2(36q(-2+3v)+q^2(-19+2s(4+s)+36v)-3(-77+4s(4+s)+72v)))}{36(-3(-1+\alpha)(1+\delta)+\alpha(3+\delta)q^2)}.$$

$[\int_0^{x_1^H} \{q(v-0)-x\}dx + \int_{x^H}^1 \{q(v-0)-(1-x)\}dx] + (1-\alpha)[\int_0^{xL}$

1

$CS_3 = \{(v-0)-x\}dx$

1

$+ \int_{x_1^L}^1 \{(v-0)-(1-x)\}dx]$

$+ \alpha\delta[\int_0^{x_1^H} \{q(v-p_2^{AH})-x\}dx + \int_{x_1^H}^1 \{q(v-p^{BH}{}_2)-(1-x)\}dx]$

$\int_0^{x_2^{AL}} \{(v-p_2^{AL})-x\}dx + \int_x^{xL}$

1

$+(1-\alpha)\delta[\{(v-p^{BR}{}_2)-(1-x)\}dx$

AL

2

xBL

2

$\int \quad +\{(v-p^{AR}{}_2)-x\}dx \; xL$

1

$+ \quad {}^1 \{(v-p^{BL})-(1-x)\}dx]2$

$\int xBL$

2

$$= -\frac{9+\square(31-2(-8+s)s+2\square(-11+(-8+s)s+6q(1+2s)))+v+(\square+\square(1+\square)(-1+q))v}{36}.$$

Thus, we can easily confirm that $\frac{\square CS_1}{\square s} = \frac{\square(2+s)(1+\square(-1+q))^2}{9+9\square(-1+q^2)} > 0$, $\frac{\square CS_2}{\square s} = \frac{\square(1-\square)(2+s)}{9} > 0$, and $\frac{\square CS_3}{\square s} = \frac{-\square\{4-s+\square(-4+6q+s)\}}{9} < 0$. QED.

Proof of Proposition 10

Let $W_\ell(s)$ ($\ell = 1,2,3$) be the W_ℓ at switching cost s. We have

$$(20+40(-1+q)+20\alpha^2(-1+q)^2)\delta s^2$$
$$+(-16+4\alpha(-1+q)+20\alpha^2(-1+q)^2-36\alpha(-1+q)q)\delta s+C_1$$

$$W_1(s) = 72((q^2-1)+1) \quad ,$$

where $C_1 = -18-22\delta-17\alpha\delta(-1+q)+5\alpha^2\delta(-1+q)^2-27\alpha\delta(-1+q)q-18\alpha(-1+q^2)$ $+72v(1+\delta)+72\alpha^2(1+\delta)(-1+q)^2(1+q)v+72\alpha(1+\delta)(-2+q+q^2)v$, and

$W_2(s) = W_3(s)$

$5(1-\alpha)\delta s^2 - 4(1-\alpha)\delta s+(-9+(-11+2\alpha)\delta)+(1+\delta)(1+\alpha(-1+q))v = \quad .$

18

Therefore, $W_\ell(s)$ s are convex quadratic functions with respect to s. Because s_1 <1 for any α and any q, SW necessarily moves from W_1 to W_2 (and then to W_3) as s increases from 0 to 1. Because $\partial W_3/\partial s||_{s=1}$ = $(1\leq-\alpha\leq)/3>0$, $W_3(s)$ is an increasing function of s when s is sufficiently high under 0 s 1.

Now, we have

-$W_3(1)-W_1(0) = \frac{4-5\alpha^2(-1+q)^2+\alpha(1-10q+9q^2)}{72((q^2-1)\alpha+1)}\delta.$

The denominator is positive for $q>1$. The numerator is

$4-5\alpha^2(-1+q)^2+\alpha(1-10q+9q^2) = 4-5\alpha^2(-1+q)^2+\alpha(q-1)(9q-1) = 4+\alpha(q-1)\{-5\alpha(q-1)+9q-1\}$.

Because $0<\alpha<1$, we have $-5\alpha(q-1)+9q-1>4q+4>0$. Therefore, the above numerator is also positive. Thus, $W_3(1)-W_1(0) > 0$. Moreover,

$W_3(1)-W_1(1) = \frac{5(1-\alpha)\alpha\delta(-1+q)^2}{8((q^2-1)\alpha+1)} > 0$. These two inequalities show that $W_3(1) > W_1(s)$ for any s such that $0\leq s\leq 1$. QED.

Proof of Proposition 11

Note that

$$\hat{\pi}^{PU\,i} = \frac{(\alpha(q-1)+1)^2\left(2\delta\left(s^2-2s+4\right)+9\right)}{18\left(\alpha\left(q^2-1\right)+1\right)}.$$

Consider whether the profit for partial UP is greater than that for BBPD. The BBPD ≤ ≤ ≤ ≤ has different profit functions for cases (i) ≤ s < s_1 , (ii) s_1 s s_2 , and (iii) s_1 s_2 s or s_2 < s_1 s (see Tables 1 and 2). In any case, s <1 holds. Case (i) corresponds to (a) of the Proposition, whereas Cases (ii) and (iii) correspond to (b). First, consider Case (i). We obtain

$$\hat{\pi}^i - \hat{\pi}^{PU\,i} = \frac{(1-\alpha)\alpha\delta(q-1)^2(1-2s)}{8\left(\alpha\left(q^2-1\right)+1\right)}.$$

Thus, we have for $0<\alpha<1$, $0<\delta\leq 1$, and $q>1$,

$s<\frac{1}{2}\Rightarrow \hat{\pi}^i > \hat{\pi}^{PU\,i}$ and $s\geq\frac{1}{2}\Rightarrow \hat{\pi}^i \leq \hat{\pi}^{PU\,i}$.

Second, consider Case (ii). Let $\Delta\pi_2(s) \equiv \hat{\pi}^i - \hat{\pi}^{PU\,i}$. We have

$$\Delta\pi_2(s) = \frac{\alpha\delta q\left(As^2+Bs+G\right)}{9\left(\alpha\left(q^2-1\right)+1\right)\left(\alpha(\delta+3)q^2+3(\delta+1)(1-\alpha)\right)},$$

where

$$\mathcal{A} = -(2\alpha(q-1)-q+2)\left(\alpha(\delta+3)q^2+3(\delta+1)(1-\alpha)\right),$$
$$\mathcal{B} = 2(2\alpha(q-1)-q+2)\left(\alpha(\delta+3)q^2+3(\delta+1)(1-\alpha)\right),$$
$$\mathcal{G} = -\left(\alpha^2(q-1)\left(\delta\left(5q^2+6q-15\right)-3\left(q^2-3q+2\right)\right)\right.$$
$$\left. +\alpha\left(\delta\left(q^3+q^2-30q+30\right)+3\left(q^3+4q^2-9q+4\right)\right)+3(\delta(3q-5)+4q-2)\right).$$

$\Delta\pi_2(s)$ is positive. We obtain

Note that the denominator of

$(\Delta\pi_2(s))$ $2\alpha\delta q(1-s)(2\alpha(q-1)-q+2)$

= .

∂s $9(q\leq 2-1\leq)+9$

The term $1-s$ is nonnegative in the range 0 s 1 , and the other terms are independent of s. Thus, the sign of this equation does not depend on s, and $\Delta\pi_2(s)$ is a monotonic function in s. Therefore, if we know that i PUi ≤ ≤ $\Delta\pi_2(s)$ >0 for both s =0 and s =1 , we can prove that $\hat{\pi} > \hat{\pi}$ for 0 s 1.

Now, we examine the case when s =0 . The sign of $\Delta\pi_2(0)$ is determined by that of G . To begin with, we examine the second-order derivative and its sign. We have

$\partial 2$G

$= 6\alpha q(3+\delta+\alpha(3-5\delta))+2(1-\alpha)\alpha(\delta+12)$. $\partial q2$

This function is linear in ≤ q, and the coefficient of q is linear in . Noticing that 0<δ1 , when δ=1 , the coefficient is $6\alpha(4-2\alpha)$ and positive if 0<α<1 . When δ=0 , the coefficient is $6\alpha(3\alpha+3)$, which is also positive. We see that the coeffi≥ -
cient is always positive. Now, because q 1 , substituting q=1 , we obtain

∂G

$= 2(21+4\delta-\alpha(3+16\delta)) > 0$,

$\partial 2q2||||q\leq=1$ 2_qG2

as 0<α<1 and 0<δ1 . Summarizing these observations, $\partial_\partial||$| is always positive.

In addition, we examine the first-order derivative:

∂G $_2$

$= 12+6\alpha+(9-25\alpha+4\alpha)\delta$.

∂q

This is a linear function of , and the coefficient can be negative depending on the ∂value of G . Assuming that the coefficient is negative, and substituting 2 ∂G δ=1 , we have ≥
∂q $= 21-19\alpha+4\alpha = (-3+\alpha)(-7+4\alpha) > 0$. Thus, ∂q >0 is shown for all q 1. Next, we examine the value of G at q=1 . We obtain

$G|q=1 = 6\geq+2(-3+\alpha)>6+2(-3+\alpha)$ because $-3\geq+\alpha<0$. Moreover, this value 2 is positive because 0<α<1 . Thus, G>0 for all q 1 . In other words, $\Delta\pi(0)$ >0 for all q 1.

Next, we examine the case when s =1 . Now, the sign of $\Delta\pi_2(1)$ is determined by that of A+B+G . We have

$A+B+G = 3(1+\alpha(q-1))(3q(1+\alpha(q-1))-\delta(3-2q+\alpha(-3+2q+q^2)))$.

Because $3(1+(q-1))$ is positive, we examine the sign of $\frac{\mathcal{A}+\mathcal{B}+\mathcal{G}}{3(1+\alpha(q-1))}$, which is a quadratic function of q. Differentiating it in q, we have

∂(A+B+G)
$= (1 - \alpha)(3 + 2\delta) + 2\alpha(3 - \delta)q.$

∂q 3(1 +(q− 1)) $\alpha(\delta+1)(q^2+q-2)v + 2\alpha^2(q-1)^2(18(\delta+1)(q+1)v + \delta(s^2+4s-17) - 18)$
$(q-1)(\delta(9q-4s(s+4)+77)+9(q+9)) + \delta(2s(s+4)+36v-43)+9(4v-5)$

This
. $\frac{\mathcal{A}+\mathcal{B}+\mathcal{G}}{3(1+\alpha(q-1))}$ ∂q > 0 $\left(\frac{A+B+G}{3(1+?(q-1))}\right)\Big|_{q=1}$ $\left(\frac{?}{} \frac{A+B+G}{3(1+?(q-1))}\right)$

Proof of Proposition 12

Let CS_{PU} be CS when firms conduct partial UP. We obtain
$\frac{A+B+G}{3(1+?(q-1))} = 3 - ?> \geq 0$
36
$\Delta ?_2(1) > 0$ for all q 1.
$-\alpha^{\partial} = 3(1+\alpha)+2(1-2\alpha)\delta$, which is always positive. This implies that ∂q >0 for all $q \geq 1$. In addition, when $q=1$, we have . Therefore, for all $q \geq 1$, which means that
Thus, we conclude that $\hat{\pi}^i > \hat{\pi}_{PU}^i$ because $\Delta\pi_2(s) > 0$ for both $s = 0$ and $s = 1$ when $q > 1$.
Finally, consider Case
$CSPU$ =.

((q2 − 1)+ 1) ≤ 1 1 ≤ ≤ 2
Note 1 ≤ that 2 ≤ BBPD 2 has 1 ≤different CS for cases (i) s s , (ii) s s s , and (iii) s s s or s < s s· ≤
First, consider Case (i). For $0<\alpha<1$, $0<\delta 1$, and $q>1$, we have

$$CS_{PU} - CS_1 = \frac{3\alpha\delta(1-\alpha)(q-1)}{8\alpha} > 0. \quad 2 \quad (q^2-1)+8$$

Second, consider Case (ii). Let $\Delta CS_2(s) \equiv CS_{PU} - CS_2$. We have
$\alpha q^3(6\alpha + \delta(-22\alpha + (2\alpha - 1)s(s+4) + 5) + 3(2\alpha - 1)s(s+4) + 15)$
$-2(\alpha - 1)\alpha q^2(\delta + (\delta + 3)s(s+4) + 21) - 3(\alpha - 1)q(-30\alpha\delta - 22\alpha + 13\delta$

$$\Delta CS_2(s) = \frac{+(2\alpha-1)\delta s(s+4) + (2\alpha-1)s(s+4)+17) + 6(\alpha-1)^2(-11\delta+(\delta+1)s(s+4)-5)}{18(\alpha(q^2-1)+1)(-3\alpha(\delta+1)+3(\delta+1)+\alpha(\delta+3)q^2)}\alpha\delta q.$$

We obtain
$(\Delta CS_2(s))$ $\alpha\delta q(s+2)(2 - 2\alpha + (2\alpha - 1)q)$ $q - 2$
= > 0 if and only if $\alpha >$.

∂s $9(q2 - 1) + 9$ $2(q - 1)$
This implies that $\Delta CS_2(s)$ is monotonically increasing or monotonically decreasing with ≤ s. Thus, we show below that $\Delta CS_2(s_1) > 0$ and $\Delta CS_2(1) > 0$ for $0 \leq < \alpha \leq < 1$,
$0<\delta 1$, and $q>1$, which implies that $\Delta CS_2(s) > 0$ for any s such that s_1 s 1 . Consider the case where $s = s_1$. We haveis a linear function of q with a positive coefficient. Then, substituting $q=1$, we

obtain (iii), where the first-time free pricing appears. In this case, the profit function is higher than that in the case where only *L*-type customer switches occur, i.e., Case (ii). This is because the price in the first period stays at zero in Case (iii) due to the nonnegative price condition. Therefore, when Case (iii) occurs, it is obvious that $\hat{\pi}^i > \hat{\pi}_{PU}^i$. QED

ΔCS where

$$_2(s_1) = \frac{\mathcal{D}\alpha\delta}{24q(\alpha(q-1)+1)^2(\alpha(q^2-1)+1)((3-3\alpha)(\delta+1)+\alpha(\delta+3)q^2)}$$

$$\mathcal{D} \equiv 18(\alpha-1)^4(\delta+1) - 16\alpha^4(\delta-3)q^7 + 4(\alpha-1)\alpha^2q^6(10\alpha(\delta-3)-3(\delta+3))$$

α $$\alpha q^5\left(10\alpha^2(\delta+3)+\alpha(5\delta-33)-3(\delta+3)\right) + (\alpha-1)^2q^3\left(4\alpha^2(92\delta+57)\right.$$

$$\delta+129)+59\delta+75) - 2(\alpha-1)^2\alpha q^4(\alpha(121\delta+51)-46\delta-78) - 12(\alpha-1)^3q^2$$

+3(−1)

−2(194

(3(6δ+5)−2(5δ+3))+9(α−1)3(2α−3)(δ+1)q.

Note that the denominator of $\Delta CS_2(s_1)$ is positive. Thus, we investigate the positivity of D . We have

∂D

5 = 360[δ{(10(α − 1)α^2

∂q^5

$$+5(\alpha-1)\alpha - 3(\alpha-1) - 112\alpha^3q^2 + 80(\alpha-1)\alpha^2q - 24(\alpha-1)\alpha q)\}$$

$$+30(\alpha-1)\alpha^2 - 33(\alpha-1)\alpha - 9(\alpha-1) + 336\alpha^3q^2 - 240(\alpha-1)\alpha^2q - 72(\alpha-1)\alpha q]$$

, which is a linear function of . We obtain

∂5D||(α3(2) α^2(8q− 3)+ 24α(3q+ 1)+ 9) > 0 and = 360 6 56q − 40q+ 5 + 21

∂q5 δ=0

∂5D||||| = 1440(2α^3(28q^2 − 20q+ 5)+ 4 α(6q+ 1)+α^2(16q− 17)+ 3) > 0 ,

∂q5 |δ=1

because $\alpha^2(16q-17)+3>0$. Therefore, we can conclude that $\frac{\partial^5 \mathcal{D}}{\partial q^5} > 0$ for all $q>1$.

Moreover, when q=1,

∂4D||||q = 24 (3 δ)+ 9α^2(75 − 31δ)+ 6α(81 − 61δ)+ 137δ+ 291) > 0 , α 4α(57 − 13

∂q4 =1

which means that $\frac{\partial^4 \mathcal{D}}{\partial q^4} > 0$ for all $q>1$. Similarly, when q=1,

∂D

3 |||| = 6(75 − 60$\alpha_4\delta$+ 59δ+ 2α3(258 − 5δ)+α2 (603 − 501δ)+ 6α(81 − 8δ)) > 0 ,

∂q3 q=1 because $75-60\alpha^4\delta>0$, which means that $\frac{\partial^3 \mathcal{D}}{\partial q^3} > 0$ for all $q>1$. We have

∂2D||q = 6(51 − 26α 3δ +α^2(159 − 5δ)+ 2α(63 − 50δ)+ 19δ) > 0 ,

___∂q2 || =1 because 51−26$\alpha^3\delta$>0 , which means that $\frac{\partial^2 \mathcal{D}}{\partial q^2} > 0$ for all q>1 . When q=1,
∂D||||q = 4(9(3 −δ)− 6$\alpha^2\delta$+α(57Dq − 13δ)) > 0 , ∂q =1
__because 9(3−δ)−6$\alpha^2\delta$>0 , which means that $\frac{\partial}{\partial}$>0 for all q>1 . Moreover, when q=1 , D = 16(3−δ) >0 . Therefore, D>0 for all q>1 . Thus, $\Delta CS_2(s_1)$ >0.

Consider the case where s =1 . We have

PU = 0, (23)
∂s 9α(q2 − 1)+ 9
∂ CS₂ 1 ≥
= (1 −α)δ(s+ 2) 0, and (24)
∂s 9
∂ CS₃ 1 ≤
= − (4 −s+(6q− 4 +s)α) 0.
∂s 9 (25)

$$\Delta CS_2(1) = \frac{2\alpha\delta q(\alpha(q-1)+1)\mathcal{F}}{3\left(\alpha\left(q^2-1\right)+1\right)\left((3-3\alpha)(\delta+1)+\alpha(\delta+3)q^2\right)},$$

where [1] ≡3(α −1)δ −α(δ −3)q^2 −(α −1)(2δ +3)q . Note that the denominator of $\Delta CS_2(1)$ and 2$\alpha\delta q$(α(q−1)+1) are positive. Thus, we investigate the positivity of

Finally, consider Case (iii). We obtain

∂CS (s+ 2)(α(q− 1)+ 1)² ≥

which imply that CS_{PU} and CS_2 are monotonically increasing with s, and that CS_3 is monotonically decreasing with s1.≤ 2

PUConsider 2 the case ≥where 1 s s . We have 2 already shown PU in Case 2 (ii) that CS >CS for all s s . Thus, when s = s , we have CS >CS . Moreover, because CS_2 =CS_3 holds for s = s_2 , we have PU CS3PU >CS≥3 at 2s = s_2 . Therefore, by Eqs. (23) and (25), we can conclude that CS >CS for all s s .

---

[1] . We have $\frac{\partial^2 \mathcal{F}}{\partial q^2}$ = 2(3−δ) >0 . Moreover, because

= (3−)+4(1−)+2>0 , >0 for all q>1 . When q=1 , F=3−>0 . Therefore, F>0 for all q>1 . Thus, $\Delta CS_2(1)$ >0.

Fq | | |q=1 F q

Consider the case where s_2 <2 ≥s_1 . Because 3 CS≥2 =1 CS_3 holds for 2 s = s_2 , it follows from Eqs. (24) and (25) that PU CS 2 CS for all ≥ 1s s (> s) . Because we have already PU 3 shown in Case (ii) that ≥ 1 CS >CS for all s s , we can conclude that CS >CS for all s s . QED.

Proof of Proposition 13

Let W_{PU} be SW when firms conduct partial UP. We obtain

$$-\delta \quad -9-9\alpha\left(q^2-1\right)+36(\delta+1)v(\alpha(q-1)+1)\left(\alpha\left(q^2-1\right)+1\right)$$
$$\left(\alpha(q-1)(2\alpha(q-1)+9q+13)+8s(\alpha(q-1)+1)^2-10(\alpha(q-1)s+s)^2+11\right)$$

W_{PU} =. 36((q2 − 1)+ 1)

(a) For $s \leq s_1$, we have $W_{PU} - W_1 = \frac{(1-\alpha)(q-1)^2(4s+1)\alpha\delta}{8(\alpha(q^2-1)+1)} > 0$.

(b) For $s_1 \leq s$, we have $SW_{PU} - SW_2 = SW_{PU} - SW_3 = \frac{(1-s)(5s+1)(-2\alpha(q-1)+q-2)\alpha\delta q}{18\alpha(q^2-1)+18} > 0$ if and only if $\alpha < (q-2)/2(q-1)$. QED.

Comparison with the case where consumers pay switching costs

For the case where consumers pay switching costs, we only provide the equilibrium price results in this subsection. The following are the equilibrium prices when Con≤diction (C) and s 1 hold.

$$\hat{p}^{A}_1 = \hat{p}^{B}_1 = \frac{(\alpha(q-1)+1)(\delta-2s\delta+3)}{3((q^2-1)\alpha+1)},$$

AL BL $(q-1)(2s+3q+4)\alpha + 2s + 4 + 3(q-1)\alpha qs$ $\hat{p}_2 = \hat{p}_2 =$ 2 ,

((q − 1) + 1)

AH BH $(q-1)(2qs+4q+3)\alpha + 2qs + q + 3 + 3(q-1)(1-\alpha)s$ $\hat{p}_2 = \hat{p}_2 =$

$$\overline{\qquad 6q((q^2-1)\alpha+1)},$$

$$\frac{((q-1)\alpha+1)(1+2s-3s)}{3((q^2-1)\alpha+1)},$$

$\hat{p}^{AR}_2 = \hat{p}^{BR}_2 =$ Compared with the case where firms pay switching costs, we find the following. First, the equilibrium price in the first period is the same as when the firm pays ≤ 1 ≤ switching costs and s s (1) , which is positive as long as the equilibrium holds. ≤ 1 We note that when firms pay switching costs, the condition s s guarantees that both types of customers switch in the second period, and no switching occurs for 1 ≤ H-type customers when $s < s$ 1 . Conversely, when consumers pay switching costs, iR2 $\hat{p}^{iR}_2$,$i = A,B$ becomes a decreasing function for ≤ s. Therefore, the relationship $\hat{p}$ $\leq \hat{p}_2^{iH}, i =$ A,B holds for all s 1 , and H-type customer switching will always occur in the second period. As a result, the price in the first period remains positive, i.e., "first-time free offer" does not appear. Second, we find that when consumers pay switching costs, H-type consumers receive discounts in the poaching prices that exceed their switching costs, whereas L-type consumers receive less than their switching costs. The poaching price becomes smaller by $\Delta p_2^{BR} \equiv \frac{(q-1)\alpha+1}{(q^2-1)\alpha+1}s$ compared with the case when firms pay switching costs. In other words, L- and H-type customers get compensation of Δp^{BR}_2 and $\Delta p^{BR}_2 q$, respectively, through price discounts on switching costs. Note that when q>1 , we have $\Delta p^{BR}_2 < s$; therefore, the L-

type customers will not be compensated for their entire switching costs. Conversely, because $\Delta p^{BR_2} q > s$, the H-type customer will be compensated by more than their switching cost For the Condition (C) case, there exist some cases in the second period where no H-type customer switches, and the price in the first period can be zero. However, because, as mentioned, Condition (C) only occurs for a very small α, in most cases it is enough to consider Condition (C) only.

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