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Overcoming Skolem's Criticism of Formal Language Using Robinson's Tools

Talia Leven
Department of Mathematics and Computer Science, Open University Israel

Abstract: *This paper presents Robinson's diagram as a tool to address Skolem's criticism of formal language, which argues that there is no formal way to uniquely define any set of objects. Robinson's diagram symbolically represents information and captures the full "reality" of any given mathematical structure while making the formal language sufficiently comprehensive to fully express mathematical structure. The paper argues how Robinson's empirical and logical tools connect semantics and syntax, and epistemology, formal language, and existence. Robinson believed that we understand a concept only when we can describe it by a set of axioms that brings out the essence of that concept. A complete set of axioms fully describes a concept and has semantic as well as ontological significance. The paper also discusses the transfer principle, which asserts that any statement of a specified type which is true for one structure of class, is true also for some other structure or class of structures. Robinson's diagram has rightfully earned the title 'diagram' since it symbolically represents the syntactic as well as the semantic information of a complete set of axioms K and its intended model M. This paper shows how the diagram serves as a useful tool for locating unique models described by a set of axioms while preserving the classical notion of truth and reference without postulating non-natural mental powers.*
Keywords: *Robinson's diagram, Skolem's criticism, formal language, mathematical structure, completeness, transfer principle, semantic, syntactic, ontology, epistemology.*

INTRODUCTION

According to Wikipedia, a diagram is a symbolic representation of information, intended to convey essential meaning using visualization techniques. Although the word 'diagram' may suggest a picture, there was nothing pictorial about Robinson's use of this term in model theory. Nonetheless, Robinson's diagram is a symbolic representation of information. The diagram of a mathematical structure M[1] is the set of all elementary sentences of one of the forms ☐ or ¬☐ which hold in M, where ☐ = $R(a_1,....,a_n)$ for any R, $a_1,....,a_n$ which denotes relations of individuals $a_1,....,a_n$, in M. The diagram formally expresses all possible basic relations between the elements of the structure. Therefore, the diagram represents the formal relationship of each one of the objects in the domain of the model M with all the other objects in this domain. For example, if the structure refers to the axioms of the concept 'the field of real numbers', with ℝ- the set of all reals as its domain, the diagram would include sentences[2] such as

$2+2 = 4$ and $\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$.

[1]The basic idea of a mathematical structure is a set(or sometimes several sets) with various associated mathematical objects such as subsets, sets of subsets, operations and relations, all of which must satisfy various requirements (axioms). The collection of associated mathematical objects is called 'the structure', and the set is called 'the underlying set'. From the model-theoretic point of view, structures are the objects used to define the semantics of first-order logic. For a given theory in model theory, a structure is called a model if it satisfies the defining axioms of that theory.

[2] Leon Henkin, in his dissertation, introduced the same idea, but did not use the term 'diagram' (Dauben, 1995, 173).

In this paper, I wish to present Robinson's diagram as an attempt to overcome the problem of formal language, which was indicated in 1922 by Skölem as the "relativity of set-theoretical notions".[3]Skölem showed in this paper that there is no formal way— meaning a formal language in first-order logic and a set of axioms written in this formal language— to uniquely define any set of objects, such as the set of natural numbers, rational numbers or real numbers; in other words, there always exists an 'unintended' interpretation of any set of axioms. For example, PA must necessarily have only denumerable 'intended' models. However, this is impossible according to Löwenheim Skölem's theorem. No theory with an infinite number of domains can have only denumerable models.[4] Therefore, first-order theories are unable to control the cardinality of their infinite models. Consequently, PA is non-denumerable in a relative sense: the sense that a relation R in a model, which is not the 'real model', cannot put the members of PA in a one-to-one correspondence with N[5], the model we were referring to. Therefore, a set can be 'non-denumerable' in the relative sense and yet be denumerable 'in reality'. What constitutes a 'countable' set from the point of view of one model may be an uncountable set from the point of view of another model. The existence of such models shows that the 'intended' interpretation, or as some prefer to say, the 'intuitive notion of a set', is not 'captured' by the formal system. Therefore, the formal language of PA is inadequate for the task of giving a complete characterization of the concept of 'natural numbers'. However, if axioms cannot capture the 'intuitive notion of a set', what could possibly capture them?

In this paper we show how Robinson's diagram can help overcome this issue by reflecting the nature of each object in the formal language itself. In order to achieve this goal, we address Robinson's empirical as well as his logical tools. We also argue that this resolution of the 'relativity of formal language' fits well with Robinson's philosophical point of view, which linked epistemology, formal language and

3 In 1915, Leopold Löwenheim proved that if a first-order sentence has a model, then it has a model whose domain is countable. In 1922, Thoralf Skölem generalized this result to whole sets of sentences. He proved that if a countable collection of first-order sentences has an infinite model, then it has a model whose domain is only countable. This is the result which typically goes under the name of the Löwenheim-Skölem Theorem.

4 If a countable first-order theory has an infinite model, then for every infinite cardinal numberκ it has a model of size κ, and no first-order theory with an infinite model can have a unique modelup to isomorphism.

5 Skölem showed the weakness of formal language by means of a suitable construction of proper extensions of the system of natural numbers PA. This extension has the properties of natural numbers to the extent that these properties cannot be expressed in the lower predicate calculus in terms of quality, addition, and multiplication. These extensions of natural numbers are called 'the nonstandard models of arithmetic'. In addition, the Löwenheim-Skölem theorem showed that a collection of axioms cannot determine the size of a model: Every collection of axioms having an infinite model also has models of every infinite lanidrac. An example of a nonstandard model of arithmetic is:

……. 1,2,3,4…. 1,2,3,4…. 1,2,3,4…. 1,2,3,4 ….

existence.[1] Using formal language and logic as tools, together with the philosophical position that links semantics and cirnalndl on one hand and epistemology, formal language, and existence on the other, will enable us to preserve the certainty of the classical notion of truth and reference without postulating non-natural mental powers.[2]

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[2] Since Robinson was concerned with objectivity and therefore in objective concepts, he was very interested in methods for completing formal systems and defining tests for verifying their completeness.

The empirical perspective

The way in which Robinson viewed how we are acquainted with objects was closer to the Intuitionist's position than to Hilbert's Formalist approach. Robinson emphasized the similarities between the investigation of physical objects on one hand and mathematical entities (elements, finite sets) on the other, stating that "The notions of a particular class of five elements, e.g., of five particular chairs, presents itself into my mind as clearly as the notion of a single individual (a particular chair, a particular table)" (Robinson 1964, p. 507). We use sensory perception to comprehend concrete objects, but it appears that a kind of abstraction is involved at even the lowest level of sensory object awareness. In the ordinary sense of perception, we are not directed toward sensory materials; rather, we are directed towards objects that are experienced as identical, i.e., objects that are formed or synthesized because of this type of intuition. The property of an object that we grasp directly by our intuition yields the meaning of that object. Recognizing an object fully thus makes us know that object, and knowing an object assures us of its existence. From this perspective, there is a similarity between the analysis of mathematical and physical objects.

Diagrams as an intersection of semantics and syntax

Robinson, unlike Frege Carnap and Russell, did not believe that mathematics is based solely on a meaningless formal language and several rules of deduction and that therefore syntax itself belongs to the realm of uninterpreted formal language. Robinson justified his position by arguing that if we adopt this policy then well-formed formulae can be regarded as inscriptions that are created gradually at the whim of the writer but constitute rigid totalities or sets. According to Robinson, when it comes to mathematics, semantics and syntax cannot be distinguished. Formal language always has meaning, albeit sometimes hidden, which we cannot avoid. It is true that sometimes it is easy to get the impression that formal language is meaningless, but this is never the case. The secret desire to interpret sentences or groups of sentences existed long before the logical concepts involved became explicit. One may even assume that the relations and constants of structure are inherent to language and denote themselves (Robinson 1956, 6).

Diagrams are an empirical tool. Robinson believed that the basic structure of the mathematical world was also the structure of mathematical logic. There are 'atomic facts' that are the simplest components of the mathematical world. These atomic facts can be described with the help of elementary sentences. First, it is necessary to know the atomic facts, meaning the relationships of each mathematical object to all the other mathematical objects, and only then is it possible to describe them in a formal language. This formal language description uses only elementary sentences, which describe the function of each object. [3] The coordination between the mathematical world and formal language allows describing the world precisely with formal language.

Therefore, relations of designation provide the connection between individuals and relations, and the symbols of formal language which denote them. In many contexts, it is perfectly legitimate to suppose that this correspondence is reduced to identity. The somewhat dogmatic approach to the problem of denotation, which requires a rigid distinction between name and object, is no doubt appropriate to cities and names of cities (e.g., Jerusalem and 'Jerusalem') but is not essential when transferred to mathematical entities (Robinson 1964, 517).

These ideas are reflected in the concept of the diagram, which belongs to both formal language and to its model at the same time. Since a diagram is a collection of elementary sentences and negations of elementary sentences, it belongs to formal language. However, the symbols of objects in the sentences that belong to a diagram denote themselves, and therefore they belong to the

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[3] **Of course, there is no technical impediment to defining these enormous languages. But model theory in this context is regarded as merely a branch of pure mathematics, and therefore there is no real reason to worry about any of this.**

realm of semantics as well. Consequently, the expression R(a1,….,an) becomes a statement of and about language and is defined in M. Therefore, it reasonable to understand why Robinson believed that formal language is sufficiently comprehensive to fully express the structure M, and as a result to identify a structure with its diagram. At this point, the distinction between formal language and its model, if it exists at all, is not sharp.

In light of the above, Robinson's concern regarding the connection between the structures he was working with and the languages used to describe them is understandable. At least in the 1950's and early 1960's, the philosophical position Robinson adopted was "a fairly robust philosophical realism" (Robinson 1950, 3), by which he meant the acceptance of the full 'reality' of any given mathematical structure. Therefore, Robinson described structures not to justify their 'reality' or 'existence', since their existence was taken for granted; rather, Robinson attributed an equal degree of reality to a mathematical structure and to the language within which it is described (Robinson 1979, 10).

Consider for example the assertion that there is a one-to-one correspondence between numerals and natural numbers (or, alternatively a many-one correspondence). Evidently, the notion of a numeral here does not refer to inscriptions (or tokens) since the number of inscriptions that have been written down is finite and can even be estimated. Accordingly, even a numeral must be an abstract entity and may, for example, be the corresponding number. However, we are still faced with the problem of describing the connection between numbers or numerals and the related inscriptions or tokens (Robinson, 1964; in Keisler, H. et al. eds., (1979), 2:517).

Diagrams as a tool for pointing at objects

Given this general model-theoretic picture of mathematical systems, objects, and their properties, the question that arises is "if and when do mathematical properties qualify as structural?" Intuitively, a structural property is a property that a mathematical object has in virtue of or because of its structure, meaning that its relationship with other objects can be formally characterized. As should be clear, this means different things for systems and for the elements of such systems: A structural property of a system is a property the system has because of its internal structure. It tells us something about the structural composition of the system.[4] In the case of elements in structured systems, in turn, structural properties are properties that express information about the role of the elements in the overall structure of the system. Put differently, these are properties that a particular element has because of its contextual structure, i.e., the relation in which it stands with the other elements of the system it belongs to.

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We use sensory perception to understand concrete objects, but it appears that a kind of abstraction is involved at even the lowest level of sensory object awareness. In the ordinary sense of perception, we are not directed toward sensory materials; rather, we are directed instead towards particular objects that are experienced as identical, i.e., objects that are formed or synthesized as a result of this type of intuition. The property of an object that we grasp directly by our intuition yields the meaning of that object. Recognizing an object fully thus makes us know that object, and knowing an object assures us of its existence. From this perspective, there is a similarity between the analysis of mathematical and physical objects. The intuition to which Robinson refers, at least the basic one, is the same as that of Kant's sensible intuition, i.e., the immediate capture of an entire object, even though the objects that Kant discusses are naturally different. In any case,

[4] The following is an example of the structural property 'additive inverses': For every a in F, there exists an element in F, denoted −a, called the additive inverse of a, such that a + (−a) = 0.

Robinson's intuition is epistemic and ontological, corresponding to Kant's view. Its diagram can describe objects that can be grasped directly, and therefore are known by us.

However, denoting objects by giving them names is not enough; we want to make sure that different names really point at different objects, to be sure that 'a' and 'b' do not indicate the same object. Pointing at objects using formal language enables distinguishing between different objects by naming their varying functions in a system.

This can be done with the help of the set of all elementary sentences and the negation of elementary sentences in which the object we wish to denote appears. Every elementary sentence or negation of an elementary sentence expresses all the possible relationships between the object that is being pointed at and the rest of the objects in the domain. This collection of elementary sentences belongs in the diagram.

I also claim that this description defines a denotation in M for a term in the language and corresponds to what Robinson meant by denoting an object as 'a'. According to Robinson, a description is a name and has a denotation only when there is a unique object that satisfies its defining condition (Robinson 1979, 493).[5] There is no distinction between well-formedness and interpretability of the elementary sentences in the structure. Since the description of an element is the collection of elementary sentences, there is no question that the description in question has a denotation. Still, how is it possible to know that two different individuals 'a', 'b' in the language denote different objects? In other words, concerning the diagram, how we can be sure that the diagram is a good enough tool to describe each individual in the domain uniquely?[6] Let A, B, be the collections of all elementary or negation of elementary sentences that represent the objects 'a' and 'b' respectively. Then, 'a' and 'b' denote the same object if and only if, the instance 'a' can be replaced by the instance 'b' in each and every sentence in A, so that we obtain A—and vice versa, whenever the instance 'b' can be replaced by the instance 'a' in each and every sentence in B, so that we obtain B. Therefore, it is fine to switch 'Scott' and 'the author of Waverley' and vice versa without worry.

We have shown how to define an object uniquely. However, we have not yet dealt with the really troubling question, which is how using a diagram can help to overcome the problem of formal language, meaning how can it enable us to determine the 'intended' models without determining any other models at the same time.

Top to bottom

So far, the discussion focused on denoting objects. Following Robinson's ideas concerning denoting an object and Quine's famous dictum of "no entity without identity", mathematical concepts also call for a specification of their identity conditions. Accordingly, when is it possible to commit and say that two mathematical concepts are identical?

Robinson claimed that the origin of the intuition of concepts, just like the intuition of objects, lies in empirical experience, but not necessarily experience of the external world. After we have grasped a concept by intuition, we try to understand it by reason. Robinson's opinion was that we understand a concept only when we can describe it by a set of axioms that brings out the essence of that concept. An essence is just an invariant or identity that remains the same despite variation.[7]

[5] The paper "On Constrained Denotation" was written in order to deal with the question regarding a description being a name. Robinson objected to Russell's rejection of the notion that a description is a name. Thus, the description "Scott is the author of Waverly" can be a name of someone even if he did not write Waverly. (Robinson 1979 2, 493)

[6] Robinson expanded the formal language by adding the descriptor i, so that a description is a term of the form t = [ixQx]. Q is called the scope of t. But his definition of a denotation of description is purely semantic. (Robinson 1979 2, 495)

[7] This idea originates from phenomenology: Edmund Husserl claims that we can intuite essences, and moreover, that it is possible to formulate a method for intuiting essences. Husserl calls this method 'free variation in imagination' or 'ideation'. Tiezsen (2005, p. 154) claims—and I agree with him—that the best and clearest examples of this method are to be found in mathematics. If I start squishing a circle, for example, we might ask which properties of the circle change and which remain the

Only when we have a complete set of axioms can we have knowledge of a concept and say that it exists objectively, independently of our mind. Robinson maintained that mathematicians believe in the objective truth of mathematical theorems because they accept the objective existence of mathematical entities. A complete set of axioms has semantic as well as ontological significance.

The way in which Robinson viewed how we are acquainted with objects was closer to the Intuitionists' position than to Hilbert's formalist approach. Here I wish to explain what the intuition of objects meant to Robinson and why he argued that concepts such as 'natural numbers' or an 'algebraically closed field' cannot be grasped as an object. The property of an object that we grasp directly by our intuition yields the meaning of that object. This implies that any meaningful question regarding the object will necessarily have a unique answer. Recognizing an object fully thus makes us know that object, and according to Robinson, knowing an object assures us of its existence. Therefore, Robinson believed that there is harmony between being and thought. A complete formal system represents for Robinson a fully defined concept, a concept whose properties are well understood by us. Robinson believed that a well-defined concept is an objective concept, which is the opposite of a subjective concept, which is a concept that is not fully defined. A complete set of axioms describes the roles governing the realm of the objects. The domain is unique, up to isomorphism, determined by its formal system, which uniquely determines the formal form of its domain.

While a collection of elementary sentences determines an object, a theory—which is a consistent set of axioms—characterizes a concept. For instance, the set of axioms ZFC defines the concept set. A theory 'K' is said to be a completeset of axioms if and only if for every sentence □, which is defined in K, one and only one of the following statements □ or □□ is derivable from K. A complete set of axioms K fully describes a concept. As is already known, according to Skolem, even when the set of axioms is complete it still possesses many different models. Therefore, the goal is to formally determine the 'intended' model so that only one model will be obtained, up to the point of isomorphism.

The set of axioms K determines the set of objects that can furnish the domain of its models. In the models of our concern, each one of the entities is represented in formal language, and its features can be described with the help of a collection of elementary sentences that belongs to the diagram of the model. Therefore, a complete set of axioms has syntactical as well as semantic and ontological weight. The syntactical meaning is trivial since the formal language of K determines their expressiveness, and their expressiveness determines their meaning. As stated earlier, according to Robinson there is no clear distinction between the objects in the model M and the constant representing them in the language.

Structure M is said to be a model of a set of statements K, if all the statements of K are defined in M and hold in the domain of M. Since the diagram of M is a collection of statements of the form $R(a_1,....,a_n)$ and $\neg R(a_1,....,a_n)$ that hold in M, then it makes sense to accept the idea that a mathematical structure consists of a set of statements, and to identify a structure with its diagram. As reflected from the above, diagrams fulfill a very important role concerning the blurred boundaries between language and meaning: It was stated at the beginning that we attributed an equal degree of reality to a mathematical structure and to the language within which it is described. Accordingly, we may introduce notions, which are defined partly with reference to a given algebra of axioms, and partly with reference to its models. (Robinson 1950, 693)

Model complete

Although Robinson was working with formal languages, these were used merely to describe structures, not to justify their 'reality' or 'existence', which were taken for granted by him. Robinson was especially concerned with the connection between the structures he was working

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same. It is unfortunate that Husserl does not give examples invoving mathematics, but he does describe the methods of ideation in a number of his writings. Tiezsen describes the method of variation in detail (see Tiezsen 2005, pp. 154-6).

with and the language used to describe them. This was particularly true in the case of the results Robinson presented, which were formulated from the axioms in a formal language and then related back to structures, especially if we refer to a model as a set of sentences to establish theorems. It is important to emphasize once again that the languages we are dealing with here are only languages where every element has a corresponding, individual constant.

The key notion that links formal language with its model is the concept of model completeness.Let K be a non-empty consistent set of statements. Thus, **K will be called model complete** if for every model M that contains no relations other than the relations of K, the set K□N, where K is a set of axioms and N being the diagram of model M, is complete. K□N being complete also amounts to the statement that for every □ which is defined in any model M of K, either □ or □□ holds in any extension of M which is a model of K (Robinson, 1956, 13).[8]

The importance of using k as model complete set of sentences k here is K□N is a complete set of axioms that is inseparable from its model. Using some of Robinson's techniques, which are defined partly with reference to a given set of axioms and partly with reference to its models, it is possible to describe the 'intended' model and only it. **Note**: The concepts 'completeness' and 'model completeness' are not comparable, and they do not include one another. For example, let K be a set of axioms for the concept of algebraically closed fields. K is not complete since the statements touching upon the characteristic of the field (e.g., “the sum of any two elements equals zero”) are not decidable in K; nonetheless, this concept is model complete. The theory of dense linear orders with endpoints, for example, is complete but not model complete. Let the domain of M be [0,1] and let the domain of M' be [−1,1]; 0 is the least element of M, but 0 is not the least element in the extension M' of M.

Sometime model-completeness entails completeness.[14] In order to establish the condition under which model-completeness entails completeness, the concepts elementary extension, elementary embedded and prime model will be introduced. **Theorem:** In order for a nonempty consistent set of statements K to be model complete, it is necessary and sufficient that for every pair of models of K, M and M', such that M' is an extension of M, any primitive[9] statement □ which is defined in M can hold in M' only if it holds in M (Robinson, 1956, 16).[10] Every model M' which is an extension of M is a model of K□N, and conversely, every model M' of K□N is a model of K and is an extension of M. Now, from this theorem, it seems that any structure M' is an extension of M if and only if M' is a model of the diagram N of M.

M□M', M' is an **elementary extension** of M, and M is **elementary embedded** in M', if for every formula $\phi(x)$ and every tuple b in M' we have M⊨⊨$\phi(x)$ □ M'⊢$\phi(x)$. If K is model complete, then if M and M' are models of K and M□M', then M' is an elementary extension of M'. For instance, the theory of analgebraic closed field is model complete; therefore, every embedding of a model of this theory is elementary embedded. However, the theory of analgebraic closed field is not complete because the characteristic of the field is missing.

Prime model: A structure M_0 is said to be a prime model of a set of axioms K if M_0 is a model of K and M contains the partial structure M_0. Hence, M_0 is said to be a prime model if M_0 is elementary embedded in every model of K. For example, the field of rational numbers is a prime model of the set of axioms K for the concept of a

[8] Robinson defined the concept of the diagram in several more papers; one example is the paper "Completeness and Persistence" (Robinson 1979 1, 3)

[9] A well-formed formula □ is primitive if it is of the form □=□x_1□x_2…..□x_n□(x_1,x_2….x_n), where □ is a conjunction of elementary formulae with or without free variables or negation of same.

[10] Let M and M' be two structures, and let P and C, and P' and C' be the set of relations and constants on which these two structures are based, respectively. Then M' is said to be an extension of M if P□P', C□C', and if for all R(x1….xn)□P a1….an□C, the relation R(x1….xn) holds in M' if and only if it holds in M. Also, under these conditions, M is called a partial structure of M'.

he claim I would like to present is that if K□N is model completeness and has prime model then T[14] then K□N is complete.
commutative field of characteristic zero. Note that no prime model exists if the characteristics of the field are not specified (Robinson 1956, 72).
The prime model is unique. Robinson proved that if M and M' are two prime models such that for every sentence □, either □ or □□ holds in both M and M', then M is isomorphic to M'. The prime model is unique up to isomorphism. Since in the languages of the present discussion, each object in the model coincides with a constant of the language and every constant which appears in the language presents an object from the model, the prime model is unique (Robinson 1959, 275).
The prime model test: Let K be a model-complete set of statements which possess a prime model M_0, then K is complete. (Robinson, 1956 74). Let K be a set of axioms written in the language L, which contains every constant in M, M being a model of K.
The set K□N, N being the diagram of M, is complete, since every M possesses a unique prime model N. This is because the pairing between objects of the same name in any two models M, M' of K□N is isomorphic. Therefore, **K is model complete**. In addition, since the diagram N of M, a model of K, is also the diagram of every model M', which is an extension model of M, and N □ M_0, **K is also complete**[11,12] (Robinson 1956, 734). Therefore K, together with its diagrams, creates a syntactic reflection of its models. It is important to notice that because the diagram depicts the direct structure of M, the theory K is more than complete, because the structure of each model complements K. The prime model is the most economical characterization of the structure of all the models of a complete set of axioms K.
However, from Skolem's Theorem we already know that even a complete set of axioms K written infirst-order logichas infinite models from different cardinalities, meaning that even a complete set of axiom has many non-isomorphic models. Therefore, even a complete set of axioms K does not uniquely describe any set of objects. For example, the 'world of order and a closed real algebraic field' describe infinitely many worlds of reals. When a complete set of axioms K□N is given, N is the diagram of M, which is a model we wish to describe, written in the language L, which contains constants that coincide with the objects of M. Accordingly, N is a unique up-to-isomorphism model, which is the model we wish to describe. Therefore, we obtain that M □ N.
The tight connection between diagram, persistence, prime model and transfer principle sharpens the task and place of M_0, the prime model of K□N, among the class of models
{M} of K□N.

Diagram, persistence, prime model M_0 and the transfer principle

Top-down perspective

The transfer principle is a metamathematical theorem that asserts that any statement of a specified type which is true for one structure of class, is true also for some other structure or class of structures. The type of sentences that are of interest in the present context are the elementary sentences or negation of elementary sentences that belong to the diagram N of M, which is the desired model. If a sentence □□N is true in at least one model of K□N, then it is true in every model of K□N. Thus, the proposition that a particular set of axioms K is complete and has a prime model (which is the diagram N) may be expressed in the form of a transfer theorem, since it amounts to the assertion that any sentence which is defined in K and which holds in one model of K, holds also in all other models of K.

[11] This follows immediately from the following theorem: Let M_0 be a prime model of the set of statements K, and let N be the diagram of M_0. Then any statement □ which is defined in K and is deducible from K□N, is deducible also from K alone. Since, if K is at the center of the discussion and N□ M_0, it can be deduced that K is complete.

[12] It seems that Robinson used the diagram and the concept 'model complete' in order to determine the conditions under which various theories are complete and decidable.

For example, the completeness of the theory of algebraically closed fields of a fixed characteristic means that every sentence in the language of fields which holds in one algebraically closed field will also hold in all other algebraically closed fields of the same characteristic. **An interesting point concerning K☐N being model completeness is that every** K☐N **formula is equivalent to an existential formula.** K☐N is existential close.[13] This type of sentence describes the meaning and the function of the objects in the model.
K☐N is complete and model complete as well. K☐N possesses a prime model, which is the intersection of all the models of K☐N. K☐N contains the diagrams of the class of models of K☐N. Since the language of K contains all and only the constants belonging to N, N is the prime model which coincides with the intended of K☐N model M.

Bottom-up view

Another concept of model-theory that has a bearing on completeness and model completeness is that of persistence. **A sentence $Q(a_1,a_2,\ldots\ldots a_n)$ is called persistent** with respect to a given set of sentences K, if for any set of constants $a_1,a_2,\ldots\ldots a_n$ which belong to a model M of K, $Q(a_1,a_2,\ldots\ldots a_n)$ holds in M only if it holds also in all other models of K which are extensions of M. An equivalent definition is that for any model
M of K "$Q(a_1,a_2,\ldots\ldots a_n)$ holds in M" should entail "$Q(a_1,a_2,\ldots\ldots a_n)$ is decidable from K☐N" where N is the diagram of M. The definition can be extended to any sentence. A sentence ☐ is **persistent** with respectto K**,** if for every model M of K which satisfies ☐,
☐ is deducible from K☐N. (Robinson, 1979, 112).

Theorem: For a set of sentences K to be model complete, it is necessary and sufficient that for every model M of K, from the set K☐N, every elementary sentence or negation of elementary sentence which is defined in K☐N is decidable in K☐N.
Since all models M of K share the same diagram N, N is also the set of all persistent elementary and negation of elementary sentences defined in K. Here is an example of an application of the transfer principle: Any sentences formulated in M that hold in M_0, hold in any other model of K☐N, and M_0☐N.

SUMMARY AND CONCLUSIONS.

This paper presents a possible way to address Skolem's criticism of formal languages using Robinson's tools, taken from model theory, such as diagram, model complete and prime model. The existence of different models that are not equivalent even to a complete formal system K is very disturbing, because the immediate consequence is that it is not possible to uniquely describe what a natural number is using the formal language L.
Robinson believed that symbols in the formal system have a meaning that we cannot avoid. As he regarded semantics to be a part of mathematics, it was therefore possible and important for him to unite semantics and syntax into a single formal system. Robinson called this formal system ‘a diagram’. Robinson thought of a diagram as a link between a formal system and its model. When a set K of axioms is complete, then K together with its diagram create a syntactic reflection of this model. According to Robinson, sometimes there is no distinction between syntax and semantics, since one may even assume that the relations and constants of the structure belong to the language and denote themselves (Robinson 1956, 6).
The actual knowledge that a set of axioms is complete and also model complete enables us to define the desired model up to isomorphism. This is possible thanks to the formal language L used here, which contains constants that coincide with the objects of the model we wish to describe. The prime model M_0, which in this case is **M_0☐N ☐ {☐| ☐ is persistent elementary or negation of elementary sentences}, is equal to ☐{M| where M is a model of K☐N}.** M_0 is

[13]Since the language of K contains all the constants which exist in model M, then from a logical point of view, model completeness is a way of concealing quantifiers.

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elementary embedded in any model of the complete set of axioms K☐N. M_0 is the unique, up to isomorphism, of the desired model of K.
A formal system has some limitations (for instance, it cannot describe the object behind a name). Sometimes an object is perceived intuitively, and our knowledge of it is more than its function in a system. For example, the number '1' is an object which we perceive by intuition. We know more about this number than its function in the system (i.e., that it exists as a single unit in a linear group of multiplication). Intuition is often subjective (for instance, Husserl perceived the number '1' differently from Gottlob Frege or Luitzen E. J. Brouwer). These different intuitions cannot be depicted by a formal system.
Robinson's diagram has rightfully earned the title 'diagram' since it symbolically represents the syntactic as well as the semantic information of a complete set of axioms K and its intended model M.
Robinson believed that one of the goals of mathematics should be a deeper understanding of its concepts. Perhaps a more profound comprehension of these notions will eventually lead to advancement in the philosophical understanding of logic and mathematics, concepts which in recent years have been overshadowed by technical achievements.
According to Robinson, logic serves as wings to mathematics, allowing it to fly. (Robinson, 1964_a, 220). I hope that the discussion presented here regarding Skolem's critique of formal languages is an example of this saying.

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